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Using Bayesian and Maximum likelihood Methods to Estimate the Survival function for NTPLD of Patients with covid-19

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Abstract **:** This article aims to estimate the survival function of patients infected with Covid-19 virus in Al-Diwaniyah city by using a new proposed formula for the New Two Parameters Lindley Distribution (NTPLD), which is one of the important continuous probability distributions that has two positive parameters(a > 0, b > 0), where the real data were fitted using Chi-Square statistics using Matlab program and the new NTPLD distribution was compared with some other distributions such as TPLD1, TPLD2 and Frechet distribution , we conclude that the new proposed Lindley distribution achieved the lowest criteria (-2LnL, AIC , AICc BIC HQIC) compared with the other distributions , which indicates that the new proposed distribution of the real data is more appropriates than the other distributions. Then, we estimated the survival function of the NTPLD distribution using two methods of estimation, the MLE method and the Bayes method. It was concluded that the MLE method is the best in estimating the reliability function (the survival function) for the new proposed NTPLD distribution**.**

Keywords - Lindley distribution, NTPLD ,Survival function, Maximum Likelihood estimation, Bayes estimation.

INTRODUCTION: The scientific advances that are achieved in the literature and the developments that occurred in the industry and in the science and technology caused the need to analyze of the phenomena , and were increased the interesting of studying the reliability, the analysis of the reliability are requires to estimate the reliability function (survival function) for the probability distributions that are used for modeling lifetime data to be studied ,for this reasons the survival function becomes an important tool in the life time studies . The survival function (reliability function) is defined as the complement of the distribution function F(t), and is denoted by S(t) or R(t).

In this paper we are used more than one formula of a Two –Parameters Lindley Distribution , Frechet distribution as well as we suggested a new formula denoted as NTPLD, the aim was to derive and estimate the survival function for the various formulas of the distributions and comparison between its to get the best one .

The survival function (or the reliability function) is defined as the probability of performing, without failure, a specific function under given condition for a specified period of time **[1]**

$$R(t) = p(T > t) = 1 - P(T \leq t) = 1 - F(t) = \int_t^{\infty} f_T(t)\, dt \quad ; t \geq 0 \qquad \dots 1$$

where $f_T(t)$: The probability density function (p.d.f)

$F(t)$: Cumulative distribution function (C.D.F)

One of the most important characteristics of it is the hazard rate function $h(t)$ which is defined as follow:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{R(t)\, \Delta t}$$

I. The Two Parameters Lindley Distribution

The Lindley Distribution is one of the continuous distributions of great importance in the study of reliability and survival theory, which has a great potential to represent the different systems that consist of complex and heterogeneous societies, as well as the high flexibility of this distribution as a failure model.

D. V. Lindley introduced a one-parameter distribution, known as Lindley distribution, given by its probability density function**: [2].**

$$f(t; b) = \frac{b^2}{(b+1)} \; (1+t) \; e^{-bt} \qquad ; t > 0 \, , \, b > 0 \qquad \dots 2$$

The cumulative distribution function (CDF) for the Lindley distribution (LD) is known by the following formula :

$$F(t; b) = 1 - \left[1 + \frac{b\,t}{(b+1)}\right] e^{-bt} \qquad ; t > 0 \, , b > 0 \quad \dots 3$$

The two-parameter Lindley distribution (TPLD1) was proposed by (Shanker et al, 2013) [**3**] as a model for studying survival times and reliability studies. It is a continuous probability distribution resulting from mixing the Gamma (2, b) distribution with the Exponential (b) distribution.

The probability density function of the Lindley distribution can be defined with two parameters by the following formula:

$$f(t;a,b) = \frac{b^2}{b+a}(1+at)\,e^{-bt}\,;\, t>0, b>0\,, a>-b \qquad \ldots 4$$

Where : a is the shape parameter and b is the scale parameter
The cumulative distribution function (CDF) for a distribution is known by the following formula:

$$F(t;b) = 1-\left[\frac{b+a+abt}{(b+a)}\right] e^{-bt} \quad ;\ t>0, b>0, a>-b \qquad \ldots 5$$

The reliability function (survival) of a Lindley distribution with two parameters was shown by the following formula:

$$R(t) = \left[\frac{b+a+abt}{(b+a)}\right] e^{-bt} \quad ;\ t>0, b>0, a>-b \qquad \ldots 6$$

And the hazard function h(t) was defined by the following formula:

$$h(t) = \frac{b^2(1+at)}{b+a+abt} \qquad \ldots 7$$

In 2016 (Shanker & Sharma) [**4**] proposed a new developed formula for the two-parameter Lindley distribution, and the probability density function of the two-parameter Lindley distribution (TPLD2) is known as the following formula:

$$\therefore f(t;a,b) = \frac{b^2}{ab+1}(a+t)e^{-bt} \quad ;t>0, b>0, ab>-1 \qquad \ldots 8$$

The following formula of the cumulative distribution function of the two parameters Lindley distribution (TPLD2) is the most commonly and recently used:

$$F(t;b) = 1-\left[\frac{1+ab+bt}{ab+1}\right]e^{-bt} \quad ;\ t>0, b>0, ab>-1 \qquad \ldots 9$$

The reliability function (survival) of the two parameters Lindley distribution (TPLD2) Lindley is shown as the following formula:

$$R(t) = \left[\frac{1+ab+bt}{(ab+1)}\right]e^{(-bt)} \quad ;\ t>0, b>0, ab>-1 \qquad \ldots 10$$

The hazard function h(t) of the Lindley distribution of two parameters (TPLD2), is defined as the following formula:

$$h(t) = \frac{1+ab+bt}{ab^2+tb^2} \qquad \ldots 11$$

II. The suggesting formula of (NTPLD)

In this paper we suggesting a new formula of the Two Parameters Lindley Distribution denote to it's as (NTPLD) that could offer a better fit to life time data. The procedure used here is based on certain mixture of two distributions Gamma of two parameters (2, b) with Exponential distribution for parameter b, by using the reciprocal of the parameter that used in the distributions in (4) as follows:

$$f(t;a,b) = p\,f_1(t) + (1-p)f_2(t) \qquad \ldots 12$$

Assuming that $p = \frac{b}{b+a}$

$$f_1(t) = \frac{1}{b}\,e^{-\frac{t}{b}} \quad , \quad t>0 \quad , \quad f_2(t) = \frac{1}{b^2}\,t\,e^{-\frac{t}{b}} \quad , \quad t>0$$

Then the new mixture distribution is denoted as (NTPLD) and defined as follows:

$$f_{NTPLD}(t;a,b) = \frac{ba}{ba+1}\frac{1}{b}\,e^{-\frac{t}{b}} + \left(1-\frac{ba}{ba+1}\right)\frac{1}{b^2}\,t\,e^{-\frac{t}{b}}$$

$$\therefore f_{NTPLD}(t;a,b) = \frac{1}{ab+1}\left(a+\frac{t}{b^2}\right)e^{-\frac{t}{b}} \quad ;t>0, b>0, ab\succ 1 \qquad \ldots 13$$

a : Shape Parameter b : Scale Parameter

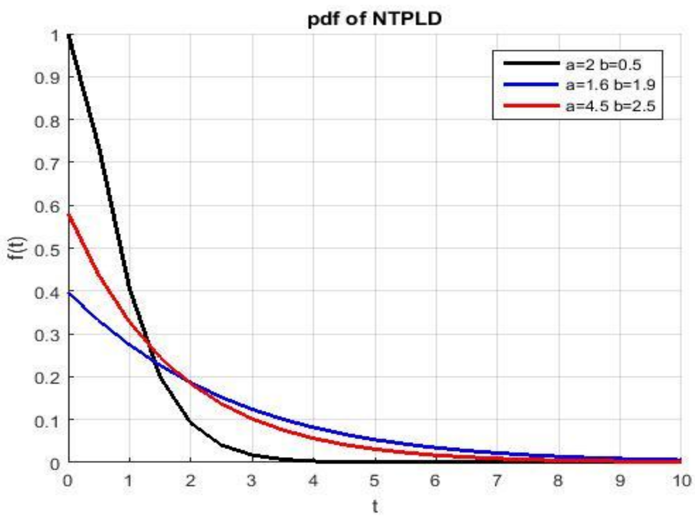

Figure 1: Plot of the p.d.f of (NTPLD) for some selected parameters a and b

The corresponding cumulative distribution function (C.D.F) of the NTPLD is:

$$F_{NTPLD}\,(t) = 1 - \left[\frac{b+t+ab^2}{b(ab+1)}\right] e^{-\frac{t}{b}} \quad , \quad x > 0, b > 0, ab > -1 \qquad \dots 14$$

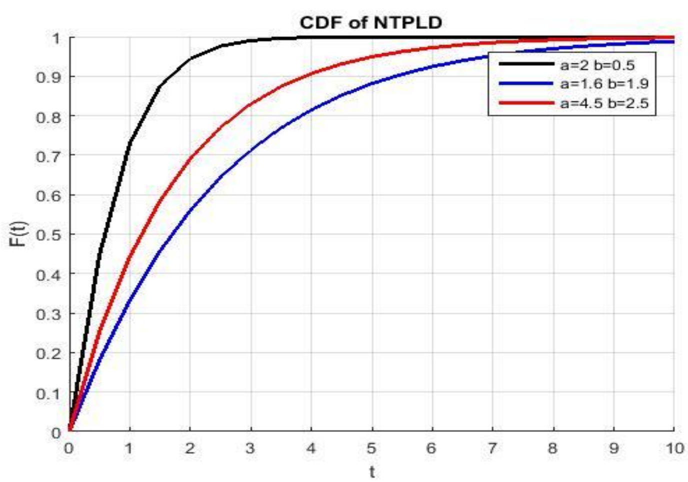

Figure 2: Plot of the cdf of the (NTPLD) for some selected parameters a and b

The $\mathbf{r^{th}}$ moments about origin of the two-parameter LD has been obtained as:

$$E(T^r) = \mu_r = \frac{1}{ab+1}\left[ab^{r+1}\,\Gamma_{r+1} + b^r\,\Gamma_{r+2}\right] \quad , r = 1,2,\dots \qquad \dots 15$$

The mean and variance of (NTPLD) respectively, are given by:

$$E(T) = \frac{ab^2 + 2b}{ab+1} \quad , \quad Var(T) = \frac{2ab^3 + 6b^2}{ab+1} - \left(\frac{ab^2+2b}{ab+1}\right)^2$$

the mode of the distribution is given by :

$$t_{mode} = b - ab^2 \qquad \dots 16$$

The (NTPLD) Reliability (survival) function is defined by the following formula:

$$R(t) = \left[\frac{b + t + ab^2}{b(ab+1)}\right] e^{-\frac{t}{b}} \qquad \dots 17$$

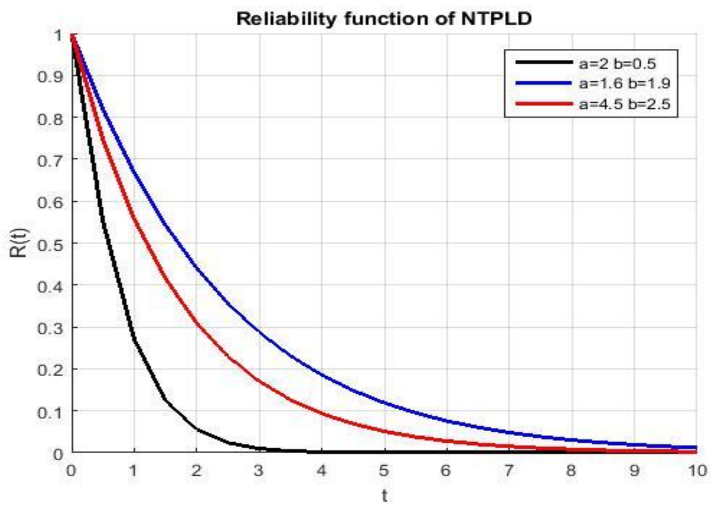

Figure 3: plot of The Reliability (Survival) Function of the (NTPLD)

The hazard rate function for the distribution (NTPLD) was defined as follows:

$$\therefore h(t) = \frac{ab^2 + t}{b(b + t + ab^2)} \qquad \dots 18$$

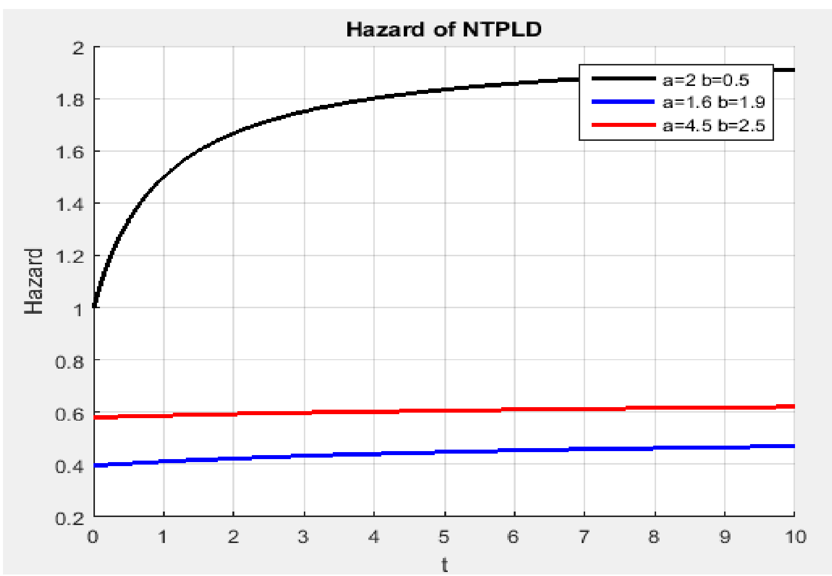

Figure 4: plot of The Hazard rate function of the (NTPLD)

III. The Frechet Distribution (FD)

The Frechet distribution is one of the lifetime models and this distribution was introduced by the French mathematician Maurice Frechet (1973-1828). It is used in failure rate modeling, which is commonly used in the study of reliability and biological studies, If the random variable y has a Weibull distribution, then the random variable x = 1/y has a Frechet distribution with the following density function is as follows : **[5]**

$$f(x, \alpha, \lambda) = \alpha\lambda\, x^{-(\alpha+1)} \exp(-\lambda x^{-\alpha}) \;\; ; \; x \geq 0 \; ; \; \alpha, \lambda > 0 \qquad \dots 19$$

We define the new distribution by the C.D.F:

$$F(x,\alpha,\lambda) = P(X \le x) = \int_0^x f(u)du = \exp(-\lambda x^{-\alpha})\,;\ x \ge 0 \qquad \dots 20$$

The Reliability (survival) function is defined by the following formula:

$$R(x) = \bar{F}(x) = \int_t^{\infty} f(x)dx = 1 - \exp(-\lambda x^{-\alpha}) \qquad \dots 21$$

The hazard rate function defined by the following formula:

$$h(t) = \alpha\lambda t^{\alpha-1} \quad t > 0, \alpha > 0, \lambda > 0 \qquad \dots 22$$

IV. The maximum Likelihood Estimation:

The method of Maximum Likelihood was presented for the first time by the researcher (Fisher, 1922), and this method is one of the important methods of estimating because it gives estimators that have good properties such as invariant, efficiency, adequacy and sometimes consistency, and the principle of this method lies in finding the parameter estimator. (Parameters) which makes the possibility function at its greatest end, and that the formulation of the possibility function was through a set of ideas that included the (Parameter) and (Consistency) and (Efficiency) in addition to (estimation). [6]

Likelihood function is a common probability function for p random variables that are used in estimating the parameters:

$$L = L(\theta_1, \theta_2, \dots, .\theta_p) \qquad \dots 23$$

And that (P) of the equations resulting from the partial derivative of the logarithm function of the possibility and equal to zero as follows:

$$\left.\begin{array}{l} \frac{\partial LogL}{\partial \theta_1} = 0 \\ \frac{\partial LogL}{\partial \theta_2} = 0 \\ \vdots \\ \frac{\partial LogL}{\partial \theta_P} = 0 \end{array}\right\} \qquad \dots 24$$

And by solving equations 24, we get the capabilities of the greatest potential (MLEs): **[7]**

If we have a random sample $(x_1, x_2, \dots, x_n)$ from the (NTPLD) distribution with a probability density function as in equation (12), then the likelihood function can be written as follows:

$$L = \prod_{i=1}^{n} f(t_i, a, b)$$

$$L = \left(\frac{1}{ab+1}\right)^n \prod_{i=1}^{n} \left(a + \frac{t_i}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b}} \qquad \dots 25$$

$$\ln L = -n\ln(ab+1) + \sum_{i=1}^{n} \ln\left(a + \frac{t_i}{b^2}\right) - \frac{\sum_{i=1}^{n} t_i}{b} \qquad \dots 26$$

To obtain the estimators of b and a, , we get the following:

$$\frac{\partial \ln L}{\partial b} = \frac{-an}{ab+1} - \sum_{i=1}^{n} \frac{2\frac{t_i}{b^3}}{a+\frac{t_i}{b^2}} + \frac{\sum_{i=1}^{n} t_i}{b^2}$$

$$= \frac{-\mathrm{an}}{\mathrm{ab}+1} - \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{2t_i}{\mathrm{b}^3\left[\frac{\mathrm{ab}^2+t_i}{\mathrm{b}^2}\right]} + \frac{\sum_{i=1}^{n} t_i}{\mathrm{b}^2}$$

$$\therefore \frac{\partial \ln \mathrm{L}}{\partial \hat{\mathrm{b}}} = \frac{\sum_{i=1}^{n} t_i}{\hat{\mathrm{b}}^2} - \frac{\hat{\mathrm{a}}\mathrm{n}}{\hat{\mathrm{a}}\hat{\mathrm{b}}+1} - \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{2\sum_{i=1}^{n} t_i}{\hat{\mathrm{b}}(\hat{a}\hat{\mathrm{b}}^2+t_i)} = 0 \qquad \ldots 27$$

$$\frac{\partial \ln \mathrm{L}}{\partial \mathrm{a}} = \frac{-\mathrm{bn}}{\mathrm{ab}+1} + \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{\mathrm{a}+\frac{t_i}{\mathrm{b}^2}}$$

$$\frac{\partial \ln \mathrm{L}}{\partial \mathrm{a}} = \frac{-\mathrm{bn}}{\mathrm{ab}+1} + \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{\frac{ab^2+t_i}{\mathrm{b}^2}}$$

$$\therefore \frac{\partial \ln \mathrm{L}}{\partial \hat{\mathrm{a}}} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\hat{\mathrm{b}}^2}{\hat{a}\hat{\mathrm{b}}^2+t_i} - \frac{\hat{\mathrm{b}}\mathrm{n}}{\hat{\mathrm{a}}\hat{\mathrm{b}}+1} = 0 \qquad \ldots 28$$

Which cannot be solved by the usual analytical methods because they are non-linear equations, so we will be solve using several iterative methods by the function (fsolve) in Mat lab to estimate $\hat{b}_{mle}$ and $\hat{a}_{mle}$. Then the estimating for the reliability or survival function of the NTLD is shown as follows:

$$\hat{\mathrm{R}}_{mle}(t_i) = \frac{(\hat{b}_{mle}+t_i+\hat{a}_{mle}\hat{b}_{mle}{}^2)}{\hat{b}_{mle}(\hat{a}_{mle}\hat{b}_{mle}+1)} \mathrm{e}^{-\frac{t_i}{\hat{b}_{mle}}} \qquad \ldots 29$$

V. Standard Bayesian Estimations

The Bayes theorem depends on the current information of the sample, which can represent the Likelihoods Function of the observations. By integrating the initial probability density function for the parameters with the maximum possibility function for the current observations, we get the Posterior probability distribution. In the Bayes method, we use a loss function, which is a function through which the loss resulting from decision-making can be measured based on the value of ($\hat{\theta}$), while the decision to be taken depends on (θ), that is, there is a difference between the parameter and its estimate. **[8]**

We need to give the initial distributions of the parameters to be estimated b and a, and according to the information available to the researcher about the initial distributions of the parameters, suppose that the initial distributions of those parameters will be as follows:

$\theta \sim$ Gamma (a_1, b_1) , $\alpha \sim$ Gamma (a_2, b_2)

Thus, the priority distribution function for each parameter is formed as follows:

$$\pi_1(b) \propto \frac{b_1{}^{a_1}}{\Gamma(a_1)} b^{a_1-1} e^{-a_1 b} \qquad \ldots 30$$

$$\pi_2(a) \propto \frac{b_2{}^{a_2}}{\Gamma(a_2)} a^{a_2-1} e^{-a_2 a} \qquad \ldots 31$$

Equations (30) and (31) represent the Prior distribution of the parameters of the Lindley distribution with two parameters (a, b) where the parameter b has gamma distribution with the Hyper-parameters (a_1) and (b_1) and the parameter has gamma distribution with the Hyper-parameters (a_2) and (b_2) based on previous experiences and experiences available to researchers.

Therefore, the joint priority is given as follows**: [8]**

$$\pi(b,a) \propto \pi_1(\theta)\pi_2(\alpha)$$

$$\pi(b,a) \propto \frac{b_1{}^{a_1}}{\Gamma(a_1)} b^{a_1-1} e^{-a_1 b} \frac{b_2{}^{a_2}}{\Gamma(a_2)} a^{a_2-1} e^{-a_2 a} \; b_1, b_2\; a_1, a_2, > 0$$

$$\pi(b,a) \propto \frac{b_1{}^{a_1} b_2{}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} b^{a_1-1} a^{a_2-1} e^{-a_1 b} e^{-a_2 a} \quad a_1, a_2 > 0 \qquad \ldots 32$$

$$\mathrm{Log}\pi(b,a) = \log\left(\frac{b_1{}^{a_1} b_2{}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} b^{a_1-1} a^{a_2-1} e^{-a_1 b} e^{-a_2 a}\right)$$

$= a_1 \log(b_1) + a_2 \log(b_2) - \log(\Gamma(a_1)) - \log(\Gamma(a_2)) + (a_1 - 1)\log(b) + (a_2 - 1)\log(a) - a_1 b - a_2 a$
…
The probability function for the observations $t_1, t_2, \dots, t_n$ is written as:

$$l(t_1, t_2, \dots\dots, t_n|\underline{\theta}) = \prod_{i=1}^{n} f(t_i, a, b)$$

$$l(t_1, t_2, \dots\dots, t_n|\underline{\theta}) = \left(\frac{1}{ab+1}\right)^n \prod_{i=1}^{n} \left(a + \frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b}} \quad \dots 33$$

The joint posterior distribution for parameters b and at the observed data can be obtained as follows:

$$h(\underline{\theta}\,|t_1, t_2, \dots\dots, t_n) = \frac{l(t_1, t_2, \dots\dots, t_n|\underline{\theta})\pi(b,a)}{\int_b^\infty \int_a^\infty l(t_1, t_2, \dots\dots, t_n|\underline{\theta})\pi(b,a)\,dbda}$$

$$= \frac{\left(\frac{1}{ab+1}\right)^n \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b}} \frac{{b_1}^{a_1} {b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} b^{a_1-1} a^{a_2-1} e^{-a_1 b} e^{-a_2 a}}{\int_b^\infty \int_a^\infty \left(\frac{1}{ab+1}\right)^n \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b}} \frac{{b_1}^{a_1} {b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} b^{a_1-1} a^{a_2-1} e^{-a_1 b} e^{-a_2 a}\,dbda}$$

$$\therefore h(\underline{\theta}\,|t_1, t_2, \dots\dots, t_n) = \frac{\left(\frac{1}{ab+1}\right)^n \boldsymbol{b^{a_1-1} a^{a_2-1}} \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b} - a_1 b - a_2 a}}{\int_b^\infty \int_a^\infty \left(\frac{1}{ab+1}\right)^n \boldsymbol{b^{a_1-1} a^{a_2-1}} \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b} - a_1 b - a_2 a}\,\boldsymbol{dbda}} \quad \dots 34$$

The Bayes estimator for the parameters of the distribution (NTPLD) under the squared loss function can be obtained as follows:

$$\hat{\underline{\theta}}_{SBayes} = E(\underline{\theta}\,|t_1, t_2, \dots\dots, t_n) = \frac{\int_b^\infty \int_a^\infty (\hat{\underline{\theta}} - \underline{\theta})^2 \left(\frac{1}{ab+1}\right)^n b^{a_1-1} a^{a_2-1} \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b} - a_1 b - a_2 a}\,dbda}{\int_b^\infty \int_a^\infty \left(\frac{1}{ab+1}\right)^n b^{a_1-1} a^{a_2-1} \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b} - a_1 b - a_2 a}\,dbda} \quad \dots 35$$

Thus, Bayes is intended for the reliability function (survival) to distribute (NTPLD):

$$\hat{R}_{SBayes} = E(R\,|t_1, t_2, \dots\dots, t_n) = \frac{\int_b^\infty \int_a^\infty \left(\frac{(b+t+ab^2)}{b(ab+1)} e^{-\frac{t_i}{b}}\right)\left(\frac{1}{ab+1}\right)^n b^{a_1-1} a^{a_2-1} \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b} - a_1 b - a_2 a}\,dbda}{\int_b^\infty \int_a^\infty \left(\frac{1}{ab+1}\right)^n b^{a_1-1} a^{a_2-1} \prod_{i=1}^{n} \left(a+\frac{t}{b^2}\right) e^{-\frac{\sum_{i=1}^{n} t_i}{b} - a_1 b - a_2 a}\,dbda} \quad \dots 36$$

We notice from equation (36) represent the Bayes estimator for the survival function of NTPLD that it does not have a closed formula to solve and is not theoretically complex. To extract the estimator of the reliability function (survival), a numerical approximation method must be used to calculate these complex integrals, which is the Lindley Approximation.

VI. Applied Aspect

Data for patients infected with the Corona virus were taken from Al-Diwaniyah General Hospital - Epidemiological Diseases Unit, which represents the times of stay until death due to Corona disease inside the hospital from the patients who were hospitalized by the number of (60) patients who fell asleep and then died as a result of infection with the Corona virus, and after appropriate work was done for the data Using the χ_c^2 statistic for good fit using Matlab program, the following results were obtained:

Table (1) Results of the data fit test results for the new two-parameter Lindley distribution

Distribution	χ_c^2	χ_t^2	Sig.	Decision
NTPLD	1.167	7.92	0.214	Accept H_0

We note from Table (1) that the calculated value of χ_c^2 (1.167) is less than the tabular value of χ_t^2 which is (7.92), and the probability value Sig = 0.214 is greater than the level of significance (0.05), which means that no rejection The null hypothesis means that the real data are distributed according to the new Lindley distribution with two parameters. Then a comparison was made between the new two-parameter Lindley distribution and the two-parameter Lindley distribution proposed by (Shanker, 2016**)** **[4]** and Weibull Distribution for the purpose of showing the best distribution when applying the real data, and the results were as in Table (2) :

Table (2) the results of the four tests of goodness of fit, which were applied to the real data

Distribution	Parameters estimation		-2LnL	AIC	AICc	BIC	HQIC
NTPLD	**2.89**	**1.12**	**12.77462**	**15.26337**	**14.45333**	**14.56556**	**5.89771**
TPLD1	**3.55**	**1.55**	**27.77013**	**29.46891**	**31.18320**	**30.01287**	**8.29668**
TPLD2	**3.15**	**1.34**	**27.35527**	**29.40746**	**31.12199**	**30.00188**	**8.26438**
Frechet	**3.54**	**1.88**	**39.127571**	**42.03262**	**43.74691**	**42.63779**	**9.04745**

Table (2) shows that the new proposed Lindley distribution achieved the lowest criteria (-2LnL, AIC , AICc BIC HQIC) compared to the other distributions (TPLD1, TPLD2, WD), which indicates that the real data fit the new proposed distribution more appropriates than the other distributions, the survival function will be estimated using the Maximum Likelihood method and the Bayesian estimation method, and the results of the estimation are shown in Table (3) below using the (Mat Lab) program :

Table (3) Survival Function Values Estimated According To the Maximum Likelihood Method and the Bayesian Method for NTPLD Distribution

t_i	S_Real	S_MLE	S_Bayes
0.10	0.98810	0.98759	0.96556
0.10	0.94605	0.94384	0.90340
0.11	0.92415	0.92110	0.89187
0.11	0.92182	0.91870	0.87619
0.12	0.91618	0.91284	0.87468
0.19	0.87009	0.86517	0.86171
0.20	0.86138	0.85617	0.81516
0.22	0.85071	0.84516	0.81439
0.22	0.84762	0.84198	0.76449
0.24	0.83782	0.83188	0.76296
0.24	0.83435	0.82831	0.76237
0.29	0.80340	0.79648	0.74696
0.34	0.77102	0.76326	0.73457
0.37	0.75648	0.74837	0.72725
0.38	0.74892	0.74063	0.69394
0.39	0.74572	0.73736	0.67807
0.39	0.74490	0.73653	0.67747

0.39	0.74249	0.73406	0.66451
0.40	0.73703	0.72848	0.66161
0.44	0.71416	0.70514	0.65711
0.49	0.68878	0.67928	0.63812
0.54	0.66288	0.65294	0.61955
0.57	0.65002	0.63988	0.60246
0.57	0.64834	0.63818	0.60188
0.63	0.61955	0.60901	0.59741
0.76	0.55732	0.54618	0.51678
0.78	0.54946	0.53826	0.49750
0.83	0.53061	0.51931	0.48004
0.84	0.52671	0.51539	0.45845
0.92	0.49404	0.48263	0.44356
0.98	0.47147	0.46003	0.43322
0.98	0.46920	0.45777	0.42189
1.00	0.46166	0.45023	0.40767
1.03	0.45278	0.44137	0.38086
1.03	0.45028	0.43887	0.37309
1.08	0.43218	0.42082	0.36862
1.09	0.43118	0.41983	0.35506
1.14	0.41474	0.40346	0.33928
1.19	0.39802	0.38684	0.31174
1.23	0.38396	0.37288	0.29345
1.36	0.34592	0.33522	0.28243
1.49	0.31079	0.30055	0.25689
1.53	0.30092	0.29083	0.22717
1.75	0.25185	0.24265	0.20133
1.76	0.24873	0.23960	0.19988
1.78	0.24627	0.23719	0.18133
1.79	0.24280	0.23379	0.16984
1.91	0.22083	0.21232	0.16541

1.99	0.20572	0.19758	0.16161
2.12	0.18572	0.17811	0.15580
2.19	0.17418	0.16691	0.11670
2.23	0.16917	0.16204	0.09650
2.30	0.15928	0.15245	0.08073
2.75	0.10910	0.10397	0.07589
3.02	0.08666	0.08241	0.06345
3.07	0.08281	0.07871	0.06229
3.26	0.07074	0.06715	0.04822
3.34	0.06599	0.06261	0.04733
3.89	0.04071	0.03850	0.02530
6.54	0.00387	0.00363	0.00310

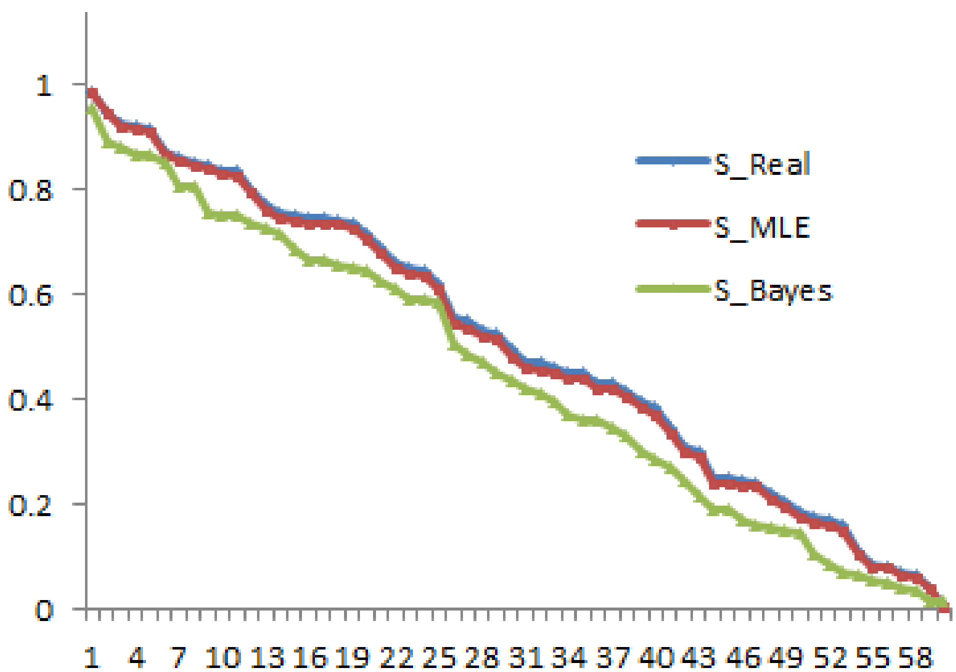

Figure (5) the survival function curve estimated according to the Maximum Likelihood Method and the Bayesian method

It is clear from Table (3) and Figure (5) that the estimated values of the survival function according to the maximum likelihood method converge to the real values. The survival function values clearly contradict time, and this corresponds to the behavior of this function being decreasing with time. And the survival function curve estimated according to the maximum likelihood method converges with the survival function curve for real data at the default parameters (a = 2.9 and b = 1).

VII. Conclusions

By analyzing the time-to-death data for people infected with Coronavirus by estimating the survival function for NTPLD distribution by using two methods of estimation the Maximum Likelihood method and the Bayesian method, it was found that the true data fit the NTPLD distribution and that the NTPLD distribution is better than the TPLD1, TPLD2 distribution. We found that the estimation of the survival function according to the Maximum Likelihood method was better than the estimation of the survival function according to the Bayesian method, as it is more close to the real data function, and the values of the survival function for the two methods are decreasing with time. We note that the longer stays of the patient in the hospital will leads to the less survive.

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