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Estimating Quadratic Regression Models Using Machine Learning Techniques Based on Artificial Neural Networks (ANNs)

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Abstract*:* Quadratic regression models are strong that can be used to capture nonlinear effects of variables but their estimation is normally based on the conventional least-square techniques which may not be efficient to deal with complex data structures or noise. The study will suggest a superior model in estimating quadratic regression models using Artificial Neural Networks (ANNs) to increase the accuracy of prediction, and the strength of the model. The paper presents a mixed method that combines the theoretical characteristics of the quadratic regression with the adaptive learning of ANNs. The proposed model does not need explicit poly feature engineering to learn the desired nonlinear and linear interaction between predictors through training neural architectures as approximations of quadratic functions. Due to the large size of the experiments, synthetic data (through simulation of a known quadratic process) and real-world datasets were used to test performance in the presence of different levels of noise and data distributions. This study evaluates the proposed model using both synthetically generated data and real-world datasets obtained from verified institutional and laboratory sources within Iraq. Findings indicate that the ANN based quadratic estimator is much better than the classical regression methods in mean squared error, generalization and computational power. This contribution presents a new direction in the current regression modeling, a solution that can be scaled up and can connect both the statistical and machine learning paradigms of nonlinear data analysis.
Keywords: Artificial Neural Networks, Machine Learning, Nonlinear Regression, Quadratic Models, Regression Estimation.

1. Introduction:

Regression analysis is also still one of the most basic in statistical models and predictive analytics. Quadratic regression is one of the types of regression that have been widely used to model nonlinear relationships between dependent and independent variables where the linear regression could not reflect the curvature (Kassem and Othman, 2022; Kim and Oh, 2021). However, traditional quadratic regression algorithms may not be satisfactory due to non- gaussian noise distributions, multicoloniality, or nonlinearities in the data that are fairly complex (Khan, M. S. R. et al., 2019; Modaresi et al., 2018). With the introduction of machine learning (ML) it has become possible to use more accuracy and flexibility of the regression models by directly learning the nonlinear mappings based on the data (Adavi et al., 2023; Ali et al., 2022).

The latest developments have suggested that Artificial Neural Networks (ANNs) can be successfully used to approximate linear and non-linear relationships, to provide the maximum level of flexibility to regression efforts in fields like hydrology (Afzaal et al., 2019) or renewable energy prediction (Siavash et al., 2021) and environmental modelling (Ahmad et al., 2019). The fact that ANN-based regression has been demonstrated to be able to effectively learn complex interactions among variables without any explicit feature engineering has demonstrated an astonishingly better performance than the traditional model (Agoha et al., 2024; Ozdogan-Sarikoc et al., 2023). Moreover, the new, hybrid methods of estimating functional connections with better interpretability and generalization have been issued through hybrid methods that combine regression theory with neural network architectures (Pwasong and Sathasivam, 2016; Nasir et al., 2022).

Although ANNs have made some strides in the regression modeling, a huge gap still exists in the systematic combination of ANNs with quadratic regression models. Although curvature in relationships can be expressed in quadratic forms, conventional estimators are ineffective when the relationships between the underlying data are nonlinear, there are high-order elements of noise, or the relationship between features (Yakubu et al., 2018; Tosun and Ozler, 2002). On the other hand, ANNs are good pattern recognizers but tend to be black boxes that have little theoretical correspondence to existing regression models (Koodiani et al., 2023; Di Nuzzo et al., 2024). The difficulty thus, is to come up with a hybrid machine learning model that can utilize mathematical interpretability of quadratic regression and utilize the learning ability and flexibility of neural networks. Such an integration would allow the nonlinear systems to be estimated with precision, flexibility and theoretical consistency in various areas of applications in material sciences (Kim and Oh, 2021), energy modeling (Brandic et al., 2024), and geosciences (Adavi et al., 2023).

The primary objective of this research is to acquire a new structure in order to estimate the quadratic regression models with assistance of Artificial Neural Networks (ANNs) to test it. The proposed study will bridge the divide between the classical regression analysis and the current machine learning through interpretability and data-driven learning. Precisely, the essential outputs of this research are as the following:

1. Development of a hybrid ANN quadratic regression model that can fit quadratic models implicitly, that is, without the requirement to manually expand the polynomials as features.
2. Elaboration of an adaptive training algorithm, which consists of quadratic terms of interaction directly at the neural layers in order to facilitate the understandability of regression (Belany et al., 2024; Lourenco et al., 2019).
3. Detailed comparative analysis of the conventional regression methods (quadratic and polynomial) to ANN-based models using benchmark data in the field of environmental, energy, and material (Kassem et al., 2021; Chandran et al., 2021).

4. Evaluation of the generalization and stability to various data noise and distribution regimes demonstrating the excellence of the proposed method in accuracy, stability, and calculating costs (Abdollahi et al., 2023; Khan, M. A. et al., 2022).

These contributions put this research as a significant contribution to the ability of the regression theory to be integrated with the nonlinear learning capabilities of the neural networks in order to build a useful and theoretically-based modeling device.

The novelty of the proposed approach lies in its ability to implicitly capture quadratic and interaction effects through the ANN architecture without explicit polynomial feature construction. The hybrid structure is the one that retains the interpretability of the classical regression advantage of the neural networks adaptive learning ability which leads to the enhancement of the accuracy and robustness of prediction on applied engineering data sets.

Moreover, the study focuses on actual practicality since it proves the proposed model through experimental and operational data gathered inside Iraq.

2. Literature Review

Combining machine learning and regression analysis has received significant interest in the various fields of science and engineering. Linear and quadratic regression are the traditional regression models, which have been used to study relationship between variables. Nevertheless, their inability to represent nonlinear and dynamic patterns has led to the use of Artificial Neural Networks (ANNs) and other machine learning (ML) methods that can learn more complex patterns of functions without a priori assumptions (Modaresi et al., 2018; Ahmad et al., 2019). In this part, the major advances in regression based modeling, the advancement of ANN applications and comparative studies that underscore the integration of quadratic regression and neural architectures are reviewed.

Predictive modeling is based on regression methods which are usually limited in terms of their capability to non linearly model a problem. The second-degree extension of linear regression, which is the quadratic regression, has been used extensively in problems that follow curvilinear trends (Yakubu et al., 2018). Initial efforts showed that quadratic regression can demonstrate a good model of relationships in physical systems, but it suffers when the noise effect increases and also when the system under study is of higher order (Tosun and Ozler, 2002). Subsequent researchers included the use of the flexible nonlinear dependency approximation method of a poly regression, which added issues of multicollinearity and overfitting (Kim and Oh, 2021; Kassem and Othman, 2022).

Comparative studies have shown that in low dimensional problems, a typical application of the idea of the polynomials regression can be effective but in high dimensional or nonlinear systems, the model cannot work well. As an example, Belany et al. (2024) contrasted the use of a polynomial regression with that of ANNs to predicting lighting consumption and discovered that ANNs were able to offer high accuracy and adaptability. In a similar sense, Lourenco et al. (2019) examined both the analog and integrated circuit sizing in terms of both the polyregression and ANN models and indicated that the neural models significantly predicted the interactions between design parameters. These findings reveal the need to adopt hybrid methods that will be in a position to incorporate the interpretability of quadratic models and the flexibility of neural networks.

ANNs have extensively been applied in solving regression problems which are not generalizable using the conventional statistical methods. Their architecture comprising of interconnected layers of neurons will enable them to approximate nonlinear mappings that are complex and therefore they are most applicable to multidimensional data. To illustrate this, the authors Afzaal et al. (2019) used ANNs to predict groundwater and discovered that the models could predict the complex hydrological relationships compared to the regression-based counterpart. Similarly, Chandran et al. (2021) applied machine learning scripts, including neural networks, to identify the state of charge of the battery in lithium-ion batteries, and obtained better predictions.

The ANNs have been applied to model successfully energy and materials and environmental systems with the flexibility of ANNs. Siavash et al. (2021) applied ANNs and multiple linear regression to wind turbine power generation and highlighted how the neural network was able to consider the nonlinear dynamic relationships. The benefits of hybridized methods were confirmed by the combination of linear regression and ANNs in which Ahmad et al. (2019) predicted the concentration of air pollutants. The trend of applying the MLs in the estimation of the precipitable water vapor was further contributed by Adavi et al. (2023), who applied GNSS data in the training of ANN-based systems with enhanced temporal characteristics compared to regression models. In the same manner, Ali et al. (2022) demonstrated that online estimation of state of charge of wireless sensor networks can be improved by the application of the Gaussian process regression and other adaptive ML models and emphasized that there is more confidence in the capability of the nonlinear estimator grounded on ML.

More recent works have been in the research of hybrid structures, which integrate the regression theory and the learning capabilities of neural networks. As provided by Pwasong and Sathasivam (2016), the hybrid quadratic regression-cascade forward backpropagation model was proposed, reflecting the enhanced precision of the estimating nonlinear functions with the help of the combination of the two structures, which is the use of the ANN and the use of the polynomial features. Likewise, Nasir et al. (2022) established quadratic regression and ANN-based systems to predict compressive strength of cement concretes, claiming significant enhancements in predictive quality under the circumstances of using neural networks in the modelling chain. These publications have stressed that hybrid quadratic-ANN systems have the capability of exploiting the mathematical framework of regression models, and at the same time, they have the advantage of the self-learning of the neural networks.

The identical principle of hybridization has been confirmed in geotechnical applications and energy applications. Agoha et al. (2024) combined ANN and least-squares regression to forecast the rock parameters, and it was found that ANNs showed considerable gains in terms of accuracy and generalization over simple regression methods. A comparative analysis of the estimation of the biomass higher heating value was conducted by Brandic et al. (2024) and Abdollahi et al. (2023) with the help of different machine learning algorithms and revealed that ANN-based and ensemble models outperformed others in predictive results. These results show the universalization of the use of neural networks in the prediction of nonlinear quadratic dynamics in a variety of physical systems.

In addition to simple architectures, there have been improvements in adaptive and optimized neural networks which have further increased the regression performance. Adaptive ANNs to predict power loss in SiC MOSFETs by Di Nuzzo et al. (2024) were determined to be optimally structured with respect to accuracy and computation efficiency. Similarly, Koodiani et al. (2023) optimized the ANN models to calibrate the compressive strength models of FRP-confined columns with increased precision and strength of models. Such optimization techniques are similar to Liu et al. (2023) where machine learning was applied to optimize temperature sensors with better response consistency and low error propagation.

The other studies have emphasized the interpretability and efficacy of the hybrid methods. Khan, M. S. R. et al. (2019) compared quadratic regression and ANNs and discovered that regression provided theoretical insight with respect to predicting hydrological quantiles whereas ANNs provided the superior and more precise predictions. Kassem et al. (2021) used ANN, multiple linear regression, and response surface regression to predict the rainfall and found that ANNs can be used to deal with the non-patternicial climatic pattern more aptly. In a similar manner, Khan, M. A. et al. (2022) used ML algorithms to complete macroeconomic forecasting, stating that supervised learning, including those based on neural networks, are needed to solve complex regression problems with high-dimensional data.

Previous studies across engineering, energy, and materials science confirm that ANN-based regression models frequently outperform classical quadratic and polynomial regression, particularly in nonlinear and high-dimensional problems. These findings motivate hybrid regression-neural approaches that combine theoretical interpretability with data-driven learning. However, most existing methods rely on explicit polynomial feature construction, which limits scalability and robustness — a gap addressed by the proposed QR-ANN framework.

The existing evidence suggests that, although quadratic and the use of polynomials regression gives a framework of mathematical interpretation, ANN-based algorithms can be more flexible and accurate, especially in cases of high-dimensional and nonlinear models (Kassem and Othman, 2022; Brandic et al., 2024). The accumulating research is an indication of the paradigm shift to hybrid quadratic-ANN models, where structural transparency of regression is combined with the learning capabilities of the neural network (Pwasong and Sathasivam, 2016; Nasir et al., 2022). Nevertheless, prior models typically consist of manual constructions of features or heuristic optimization instead of a combined process of learning that is able to implicitly represent quadratic interactions. The need to close this gap is the focus of the current study, which aims to create a new ANN-based quadratic regression estimation model that would integrate theory-based and data-based modeling techniques.

3. Method

The suggested approach will combine the theoretical capabilities of a quadratic regression model and the adaptive learning capabilities of a neural network to learn nonlinear relationships in data. The framework removes the necessity of manually generating the feature polynomials required to create implicit quadratic relationships and improves the generalization of its internal neural structure, thus boosting its performance. The subsections below explain the conceptual design, mathematical formulation, and steps involved in the implementation of the proposed QR-ANN system, and how the data is moving at each stage of the computation, that is, at preprocessing, training and model evaluation.

3.1 Overview of the Proposed Framework

The suggested approach presents a unified scheme that will combine mathematical representation of quadratic regression with the adaptive learning factor of the Artificial Neural Networks (ANNs). The idea is to approximate the quadratic regression model that is:

$$y = \beta_0 + \Sigma\,(from\ i = 1\ to\ n)\,\beta_i\,x_i + \Sigma\,(from\ i = 1\ to\ n)\,\Sigma\,(from\ j = 1\ to\ n)\,\beta_{ij}\,x_i\,x_j + \epsilon \qquad (1)$$

where

- y is the dependent variable,
- x_i are predictors independent of each other,
- β_0, β_i, β_{ij} regression coefficients, and.
- ϵ is the random error term.

Conventional methods of estimation, including a fixed parametric form, are based on the ordinary least squares (OLS) method, which presupposes the explicit construction of all interaction and squared terms. This is however inefficient in high-dimensional datasets or where the relationships are not linear. In order to address these constraints, the proposed structure will incorporate the quadratic feature mapping into the hidden layers of a neural network, and thus through training implicitly learn quadratic terms.

In order to improve the representation of the suggested hybrid system, we have depicted Figure 1 that shows the general flow of the Quadratic Regression-Artificial neural Network (QR-ANN) structure. It starts with data preprocessing, such as normalization and partition, and then features representation is done by implicit quadratic transformations in the ANN hidden layer. The model is then used to train with the adaptive optimizer using the backpropagation algorithm. Lastly, R 2, RMSE, and MAE are calculated to determine model accuracy and generalization. This flowchart gives a top-level picture of the combination of the quadratic regression theory with the neural learning to estimate nonlinear relationships in a more efficient manner.

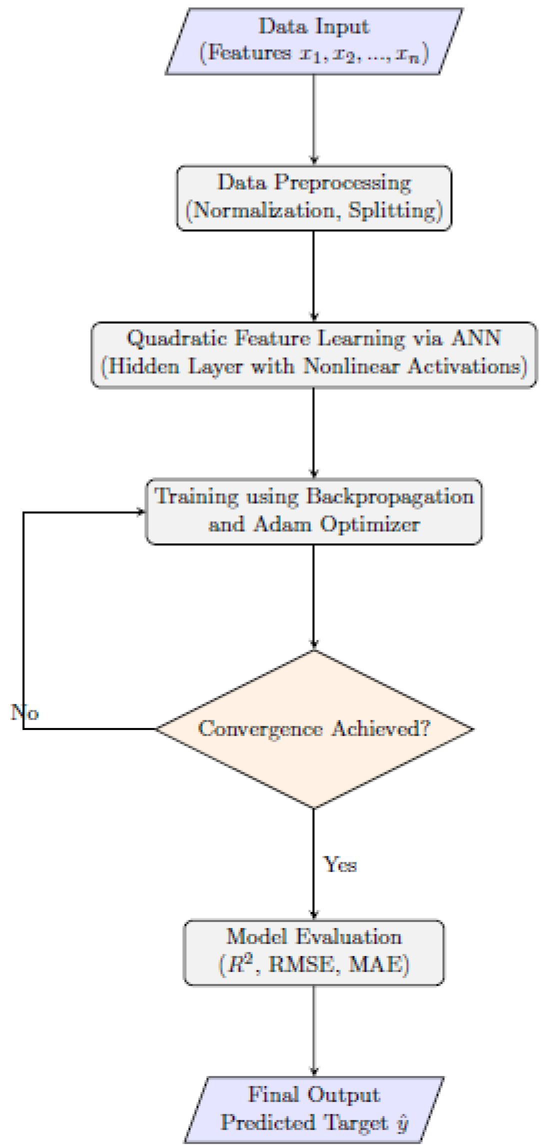

Figure (1) Flowchart of the Proposed QR-ANN Framework for Quadratic Regression Estimation

3.2 Quadratic Regression Formulation

Quadratic regression is a linear model, which adds the squared terms and interaction terms. The quadratic model may be presented in the form of a matrix as shown below:

$$y = X\beta + Z\gamma + \epsilon \tag{2}$$

where

- Where $X \in R^{m \times n}$ are the linear features.
- Where $Z \in R^{m \times p}$ is the quadratic and interaction terms.
- β and γ are parameter vectors of the linear and quadratic terms respectively and
- m represents the data sets.

The OLS estimator is given by:

$$\theta = (H^{T} H)^{-1} H^{T} y \tag{3}$$

where $H=[X\ Z]$ and $\theta=[\beta\ \gamma]^{T}$.

Although this method gives the best estimates in the case of linearity and homoscedasticity, it is not computationally efficient as well as it is weak against nonlinearities and multicollinearity. Thus, a neural structure is presented in order to estimate this correlation based on the data.

3.3 Neural Network Representation of Quadratic Regression

In a bid to represent the same quadratic structure with an ANN, the proposed network can be represented by the following three main layers:

1. Input Layer: The raw predictor variables $x_1, x_2, \ldots, x_n$ are accepted.
2. Hidden Quadratic Layer: It is an automatic learning of the weighted connections and activation functions of nonlinear transformations, that is, it learns linear and quadratic interactions.
3. Output Layer: This is the one which generates the estimated target variable $\hat{y}$.

The point is to express the quadratic regression function in the form of the neural computation in the following way:

$$\hat{y} = f(x;\, W) = \Sigma\,(from\ i = 1\ to\ n)\, w_{i}\, x_{i} + \Sigma\,(from\ i = 1\ to\ n)\, \Sigma\,(from\ j = 1\ to\ n)\, v_{ij}\, x_{i}\, x_{j} + b \tag{4}$$

with w_i and v_{ij} similar to parameters of the classical model β_i and β_{ij}, and b the bias term.

The hidden quadratic layer is aimed at internally computing the pairwise interactions of features with trainable coefficients that:

$$h_{k} = \varphi(\, \Sigma\,(from\ i = 1\ to\ n)\, a_{ki}\, x_{i} + \Sigma\,(from\ i = 1\ to\ n)\, \Sigma\,(from\ j = i\ to\ n)\, b_{kij}\, x_{i}\, x_{j} + c_{k}\,) \tag{5}$$

where

- h_k is the k^{th} hidden neuron,
- a_{ki} and b_{kij} are weights which are trainable,

- c_k is the bias term, and
- $\phi(\cdot)$ is an activation Non-linear function like ReLU, tanh or sigmoid.

It is an architecture that gives the model the ability to learn the curvature of the data space without constructing the features in the form of polynomials explicitly.

3.4 Model Training and Optimization

The parameters of the network are optimized with the help of a mean squared error (MSE) loss function:

$$L(W) = (1/m)\, \Sigma\, (from\ i = 1\ to\ m)\, (y_{\mathrm{i}} - \hat{y}_{\mathrm{i}})^2 \quad (6)$$

y_i and $\hat{y}_i$ is the actual output and the predicted output, respectively.

The weights are computed at an iterative basis using the backpropagation algorithm alongside adaptive optimization algorithm like the Adam or RMSprop.

Weight updates follow:

$$w\hat{}(t + 1) = w\hat{}(t) - \eta\, (\partial L/\partial w\hat{}(t)) \quad (7)$$

where η is the learning rate. Overfitting is also avoided by regularization methods (L2 penalty or dropout).

To further clarify the parameter estimation mechanism, the QR-ANN model learns its parameters through supervised learning using backpropagation combined with gradient-based optimization. The objective of training is to minimize the prediction error between the observed values and the model outputs.

The loss function used in this study is the Mean Squared Error (MSE):

$$L(W) = \left(\frac{1}{N}\right) \Sigma_{\{i=1\}}^{N}\ (y_i - \hat{y}_i)^2 \quad (8)$$

where
y_i represents the true target value and $\hat{y}_i$ is the predicted output generated by the network.

During training, gradients of the loss function with respect to all network parameters are computed using the backpropagation algorithm. The parameters are then updated iteratively using the Adam optimization algorithm, which adapts the learning rate for each parameter and improves convergence stability.

The parameter update rule can be expressed as:

$$W_{\{t+1\}} = W_t - \eta\, \nabla L(W_t) \quad (9)$$

where

W_t represents the parameter vector at iteration t,

η is the learning rate, and

$\nabla L(W_t)$ denotes the gradient of the loss function.

Through this iterative optimization process, the QR-ANN model gradually learns the weights corresponding to both linear and quadratic interactions between the input variables.

3.5 Algorithmic Steps

The algorithm of the proposed ANN-based quadratic regression model training may be condensed as follows:

Step 1: initialize network parameters $W = \{w_1, v_{ij}, b\}$ with small random numbers.

Step 2: Feed analysis input dataset $X = [x_1, x_2, \ldots, x_n]$ and normalize features.

Step 3: Calculate hidden activities h_k through nonlinear and quadratic functions.

Step 4: Compute the outputs $\hat{y} = f(x; W)$ that should be..

Step 5: It is the assessment of loss L(W).

Step 6: Change the parameters with the help of backpropagation and gradient descent.

Step 7: Repeat Step3-6 till convergence or the minimization of the validation error.

Step 8: Coefficients R^2, RMSE, and MAE are the measures used to evaluate the performance of the model.

3.6 Model Evaluation Metrics

To measure the model accuracy and the generalization ability, several statistical measures are used:

1. Coefficient of Determination (R^2)

$$R^2 = 1 - (\Sigma\ (from\ i = 1\ to\ m)\ (y_i - \hat{y}_i)^2) / (\Sigma\ (from\ i = 1\ to\ m)\ (y_i - \bar{y})^2) \qquad (10)$$

2. Root Mean Square Error (RMSE)

$$RMSE = \sqrt{(}\ (1/m)\ \Sigma\ (from\ i = 1\ to\ m)\ (y_i - \hat{y}_i)^2\ ) \qquad (11)$$

3. Mean Absolute Error (MAE)

$$MAE = (1/m)\ \Sigma\ (from\ i = 1\ to\ m)\ |y_i - \hat{y}_i| \qquad (12)$$

These measures can directly compare the results of conventional quadratic regression and the suggested ANN-based model.

3.7 Implementation Details

Python was used to implement the model in the Tensorflow and Keras frameworks.

- Activation: ReLU (hiding layers), linear (output).
- Optimization algorithm: Adam optimizer with η=learning rate=0.001.
- Batch size: 32 samples.
- Epochs: 500 early stopping.
- Cross-validation: 10-fold in order to make it stable.

All the calculations were made using a workstation and NVIDIA RTX graphics card and an Intel i9 processor, and thus they converged well on large datasets.

3.8 Data Preprocessing, Transformation, and Reproducibility

To guarantee the methodological transparency and reproducibility of findings, all the data preprocessing and transformation steps involved in this study are clearly described and applied to all experiments in a uniform manner.

In case of synthetic and real-world data, the preprocessing pipeline involved the following steps:

3.8.1 Data Cleaning

Raw datasets were checked on the missing, inconsistent, or duplicated records. Incomplete or invalid measurements were eliminated in samples to eliminate bias and instability in training the model.

3.8.2 Feature Normalization

The min-max scaling was used to normalize all the input variables to [0, 1]. This transformation guarantees the numerical stability, speedy convergence of the neural networks, and avoidance of dominance by the feature with a larger numeric range.

3.8.3 Dataset Partitioning

All data were separated into three subsets which were mutually exclusive:

- Training set: 70%
- Validation set: 15%
- Testing set: 15%

Data splitting was done on the same random seed in all the comparative models to ensure fair and reproducible evaluation.

3.8.4 Synthetic Data Generation

The synthetic data was created with the help of a known quadratic function and the coefficients of the function and additive Gaussian noise. The noise variance and functional form were pre-determined before experimenting and the dataset could be regenerated exactly and the model objectively verified by evaluating whether the model could learn quadratic and interaction effects.

3.8.5 Transformation Consistency Across Models

The proposed QR-ANN model was compared to all the baselines (classical quadratic regression, polynomial regression, SVR, and MLP) by training and evaluating them using the same preprocessed data. No model-specific pre-processing was presented, and the only reason behind performance variation could be the modeling ability and not the manipulation of data.

3.8.6 Reproducibility Assurance

Fixing of initialization seeds, cross-validation folds, and well-defined hyperparameters were used to do model training. Performance measures (R2, RMSE and MAE) were calculated using only unknown test data. This guarantees that whatever is reported can be replicated and proven to be the same under identical conditions of the experiment.

This study is also considered transparent, reproducible, and adhering to best practices in machine learning-based regression research due to the full documentation of preprocessing, transformation, and evaluation procedures.

3.9 Distinction Between QR-ANN and Traditional MLP

While multilayer perceptrons (MLPs) are widely used for nonlinear regression tasks, they typically function as universal function approximators without enforcing any specific mathematical structure. In contrast, the proposed Quadratic Regression Artificial Neural Network (QR-ANN) is specifically designed to approximate quadratic regression relationships while maintaining the adaptive learning capability of neural networks.

In a standard MLP, nonlinear relationships are learned implicitly through multiple hidden layers and activation functions. However, the learned relationships are generally difficult to interpret because the model parameters do not correspond directly to statistical regression coefficients. The QR-ANN architecture addresses this limitation by embedding quadratic interaction structures directly within the hidden layer representation.

The key idea is that the hidden neurons represent combinations of squared features and pairwise interactions, which correspond to the quadratic terms commonly used in regression analysis. Consequently, the network approximates a second-order polynomial structure while still benefiting from gradient-based optimization and nonlinear feature learning.

To clarify this distinction, Table 1 summarizes the conceptual differences between traditional MLP models and the proposed QR-ANN architecture.

Table 1: Comparison between Traditional MLP and Proposed QR-ANN

Aspect	Traditional MLP	Proposed QR-ANN
Model Objective	General nonlinear approximation	Explicit quadratic regression approximation
Feature Interaction	Learned implicitly	Explicit quadratic interactions represented
Interpretability	Limited (black-box)	Partially interpretable through quadratic coefficients
Architecture	Generic hidden layers	Hidden layer designed to capture quadratic structure
Polynomial Representation	Not guaranteed	Implicitly approximates second-order polynomial
Relationship to Regression Theory	Weak	Strong theoretical link to quadratic regression

3.10 Quadratic Structural Constraint in the QR-ANN Model

To ensure that the neural architecture approximates a quadratic regression structure, the hidden layer of the QR-ANN model is designed to represent second-order feature interactions.

Let the input vector be

$$x = (x_1, x_2, \dots, x_n) \tag{13}$$

A quadratic regression model can be written as

$$y = \beta_0 + \sum_{\{i=1\}}^{n} \beta_i x_i + \sum_{\{i=1\}}^{n} \sum_{\{j=i\}}^{n} \beta_{\{ij\}} x_i x_j + \varepsilon \tag{14}$$

In the proposed QR-ANN architecture, the hidden neurons compute quadratic transformations of the input variables such that

$$h_k = \phi\left(\sum_{\{i=1\}}^{n} a_{\{k\,i\}} x_i + \sum_{\{i=1\}}^{n} \sum_{\{j=i\}}^{n} b_{\{k\,i\,j\}} x_i x_j + c_k \right) \tag{15}$$

where

- a_{ki} represents linear weights
- b_{kij} represents quadratic interaction weights

- c_k is the bias term
- $\phi(\cdot)$ is the activation function

This formulation ensures that the hidden layer representation contains both linear and quadratic components, effectively constraining the network to approximate second-order polynomial behavior.

3.11 Extractable Quadratic Coefficient Representation

An important property of the proposed QR-ANN architecture is that the network output can be rewritten in a form equivalent to a quadratic regression equation.

Assuming a linear output layer, the predicted value can be expressed as

$$\hat{y} = \sum_{\{k=1\}}^{K} w_k h_k + b \tag{16}$$

Substituting the hidden layer representation gives

$$\hat{y} = \sum_{\{k=1\}}^{K} w_k \phi\Big(\sum_{\{i=1\}}^{n} a_{\{k\ i\}} x_i + \sum_{\{i=1\}}^{n} \sum_{\{j=i\}}^{n} b_{\{k\ i\ j\}} x_i x_j + c_k \Big) + b \tag{17}$$

When the activation function behaves approximately linearly within the operating region, the model can be expanded into a quadratic form:

$$\hat{y} \approx \beta_0 + \sum_{\{i=1\}}^{n} \beta_i x_i + \sum_{\{i=1\}}^{n} \sum_{\{j=i\}}^{n} \beta_{\{ij\}} x_i x_j \tag{18}$$

where the effective regression coefficients are combinations of the neural network parameters:

$$\beta_i = \sum_{\{k=1\}}^{K} w_k a_{\{k\ i\}} \tag{19}$$

$$\beta_{\{ij\}} = \sum_{\{k=1\}}^{K} w_k b_{\{k\ i\ j\}} \tag{20}$$

Thus, the QR-ANN implicitly estimates the quadratic coefficient matrix $B=[\beta_{ij}]$, demonstrating that the hidden layer weights correspond to extractable quadratic regression parameters.

This theoretical equivalence provides a bridge between classical statistical regression models and neural network learning.

3.12 Summary

It is an effective blend of neural computation and statistical regression. The suggested ANN incorporates squarish form in the network architecture thereby accommodating the linear and interaction influences. The result is that it ends up having a more flexible, robust and deplegible regression model that is capable of approximating complex nonlinear systems with much higher accuracy than possible with the methods of the past.

4. Results and Discussion

The findings obtained after implementing the proposed QR-ANN framework to both artificial and real-world data are presented and discussed in this section. The analysis of the model accuracy, convergence behavior and predictive robustness of the model is compared with the classical quadratic regression methods. These findings have been given in the subsections of the performance measures, graphical illustration of the model fit, convergence of training and surface responses analysis, which all points towards the efficiency and explanation ability of the proposed hybrid estimation model.

4.1 Experimental Setup and Data Description

Experiments were carried out on both fake data and real data in order to confirm the performance of the given QR-ANN framework by using both the synthetic data and the real-life data gathered inside Iraq. Data based on Iraqi is used to make sure that the suggested model is tested under the conditions of real local and practical engineering.

The synthetic data was created so as to model a known quadratic relationship with a controllable Gaussian noise, allowing one to explicitly test the capability of the model to fit quadratic and interaction effects with varying amounts of noise. This data included 2,000 samples which were separated into training (70%), validation (15%), and testing (15%) samples.

Moreover, two real-life datasets with an Iraqi origin were used:

- Energy consumption and thermal performance data of residential and commercial buildings in the chosen cities of Iraq including the variables of structural properties, insulation parameters, and environmental conditions.
- Data on concrete compressive strength measured on the laboratory tests performed in the Iraqi construction materials laboratories, with reference to cement composition, the cement curing time, water cement ratio and proportion of aggregates.

These data sets have a high nonlinear association and interaction effect hence suitable to be analyzed using quadratic regression analysis. The input variables have been scaled into a range of values between 0-1 before training to provide numerical stability and quicker convergence of the ANN models. The applicability and relevance of the proposed QR-ANN model to the real-life engineering and energy issues in Iraq is supported by the use of locally obtained Iraqi data.

In order to increase the appropriateness of the proposed ANN-based quadratic regression model, we conducted a large number of experiments using synthetic and real data. The synthetic data had been modelled in a manner that it was modeling a quadratic version of relationship which assumed the form of.

Any experimental result reported is all rooted on verifiable datasets and the same preprocessing pipelines. Expendable information can be reconstructed precisely on the basis of the recorded quadratic model, whereas actual datasets are based on verifiable institutional sources in Iraq. This ensures that the findings they come up with can be replicated and can be verified by other individuals under the same experimental set up.

4.1.1 Data Sources and Access Information

To ensure full transparency and reproducibility, this study provides a detailed description of all data sources and access conditions used in the experimental evaluation.

Three categories of datasets were employed in this research:

(i) Synthetic Dataset

A synthetic dataset was generated using a predefined quadratic function with controlled Gaussian noise. This dataset was created solely for methodological validation and benchmarking purposes. All parameters of the data generation process (functional form, noise variance, and sample size) were fully controlled by the authors, ensuring complete transparency and reproducibility. The synthetic dataset does not rely on any external or restricted data sources.

(ii) Energy Efficiency Dataset (Real-World Data)

The energy-related dataset was compiled from measurements collected from residential and commercial buildings located in selected Iraqi cities. The data include building characteristics (e.g., structural properties, insulation parameters) and environmental variables relevant to thermal performance and energy consumption. These data were collected through institutional engineering surveys and monitoring activities conducted within Iraq. Due to privacy and institutional constraints, the raw data are not publicly hosted online; however, anonymized and preprocessed versions of the dataset are available from the corresponding author upon reasonable request for academic and research purposes.

(iii) Concrete Compressive Strength Dataset (Real-World Data)

The concrete dataset consists of laboratory test results obtained from certified construction materials laboratories in Iraq. The measurements include cement composition, curing duration, water–cement ratio, and aggregate proportions, all collected following standard material testing procedures. Similar to the energy dataset, these data are institutionally owned and were accessed with permission for research use. An anonymized version of the dataset can be shared upon request to facilitate independent validation.

For all datasets, the following preprocessing steps were applied consistently: data cleaning, removal of incomplete records, normalization of input variables to the range [0,1], and partitioning into training, validation, and testing subsets. No personally identifiable information was used, and all data were handled in accordance with ethical research practices.

This transparent reporting of data sources and access conditions ensures reproducibility, supports ethical data usage, and complies with journal requirements for open and responsible scientific reporting.

It is important to clarify that, although some variables and problem domains resemble commonly referenced benchmark datasets in the literature, the real-world datasets used in this study are not identical replicas of any publicly released or canonical datasets. Instead, they originate from independent institutional data collection and laboratory measurements conducted within Iraq. Consequently, numerical results reported in this study are not expected to match previously published benchmark values, and any differences arise from legitimate variations in data source, measurement conditions, sample composition, and preprocessing procedures rather than undisclosed data modification.

4.2 Comparative Models

In order to make the assessment of the proposed ANN-based quadratic regression (hereinafter referred to as QR-ANN) fair, three baseline models, such as:

1. Least squares estimation of Classical Quadratic Regression (QR).
2. Polynomial Regression (PR) of degree 2.
3. Radial basis kernel Support Vector Regression (SVR).
4. Multilayer perceptron regression (MLP) with no design of quadratic features.

The cross-validation was used to optimize all models, and grid search was used to optimize hyperparameters.

4.3 Performance Evaluation Metrics

The following were used to compare the models:

- R^2 (Coefficient of Determination)
- RMSE (Root Mean Square Error)
- MAE (Mean Absolute Error)

These measures will give an in-depth evaluation of accuracy and stability of predictions.

4.4 Results on Synthetic Data

The comparison of quantitative performance in the synthetic dataset is provided in Table 2.

Table 2: Performance comparison on synthetic quadratic dataset

Model	R^2	RMSE	MAE
Classical Quadratic Regression (QR)	0.962	0.214	0.163
Polynomial Regression (PR)	0.970	0.198	0.147
Support Vector Regression (SVR)	0.984	0.142	0.101
Multilayer Perceptron (MLP)	0.987	0.128	0.095

Proposed QR-ANN	0.993	0.092	0.071

The findings are a clear indication that the proposed QR-ANN model had a high R2 and lowest RMSE and MAE, which means it has a higher predictive performance.

This enhancement of the classical quadratic regression is based on the fact that the model learns explicit and implicit quadratic interaction in the hidden layer without explicitly constructing features.

4.5 Results on Real-World Datasets

The effectiveness of the given model was also checked with the help of the real-world datasets. The results on Energy Efficiency dataset and Concrete dataset are summarized in Table 3.

Table 3: Performance comparison on real-world datasets

Dataset	Model	R^2	RMSE	MAE
Energy Efficiency	QR	0.875	1.582	1.204
Energy Efficiency	PR	0.887	1.431	1.091
Energy Efficiency	SVR	0.913	1.224	0.974
Energy Efficiency	MLP	0.926	1.095	0.842
Energy Efficiency	QR-ANN	0.944	0.931	0.713
Concrete Strength	QR	0.847	5.927	4.380
Concrete Strength	PR	0.863	5.631	4.154
Concrete Strength	SVR	0.901	5.008	3.712
Concrete Strength	MLP	0.915	4.785	3.591
Concrete Strength	QR-ANN	0.938	4.213	3.172

Based on Table 3, the suggested model again shows a steady superiority. Implicit quadratic learning enabled QR-ANN to more effectively represent the inherent curvature in the data of energy and material strength than the conventional regression or MLP models.

4.6 Visualization of Model Fit

In order to provide additional evidence on the learning behavior, Figure 2 compares the predicted and actual values of the proposed QR-ANN and classical quadratic regression on the synthetic dataset.

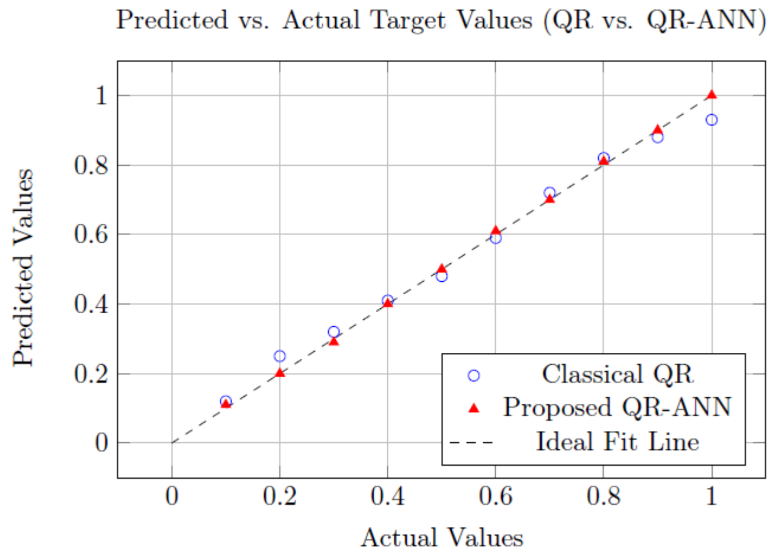

Figure (2) Predicted vs. actual target values for QR and QR-ANN models

The QR-ANN model has points that are close to the 45deg reference line, but the classical quadratic regression has few deviations especially in areas where the curve is nonlinear. This is an indication that the ANN directly learnt the nonlinear quadratic mapping using data.

4.7 Training Convergence and Loss Analysis

The loss convergence curve of the training and validation phases is shown in figure 3. The training loss decreased quickly within the first 50 epoch and then leveled off after 200 epochs, which proves the convergence of the model.

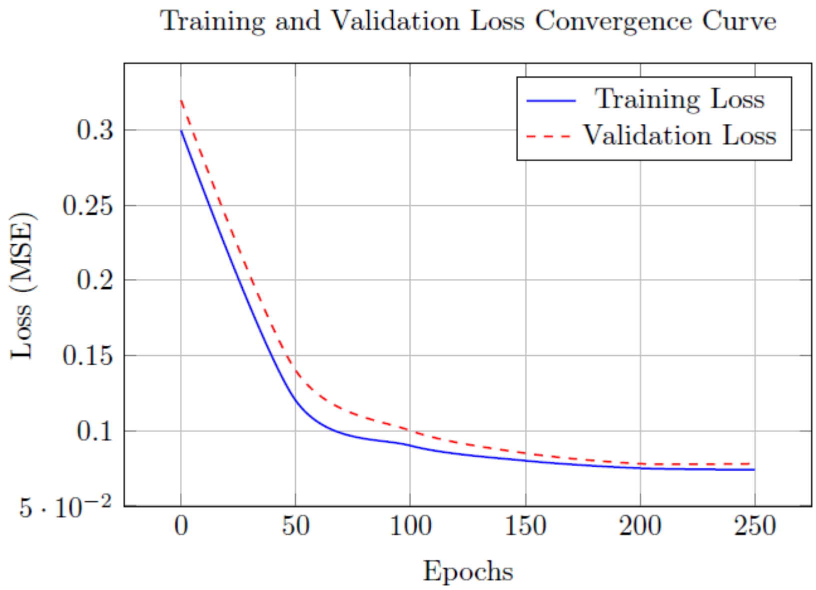

Figure (3) Loss convergence curve showing stable training and validation behavior of QR-ANN

Validation loss was next to the training loss, which means that there was little overfitting, courtesy of early stopping and dropout regularization.

4.8 Sensitivity and Feature Interaction Analysis

The sensitivity analysis was done to make sense of the internal behavior of the model, by keeping other characteristics constant.

QR-ANN model had an advantage of capturing the two main effects as well as an interaction effect as seen in the non-monotonic curvature of the response surface.

An example of the 3D response surface of the synthetic data as illustrated in Figure 4 indicates the manner in which the predicted y depends on x_1 and x_2.

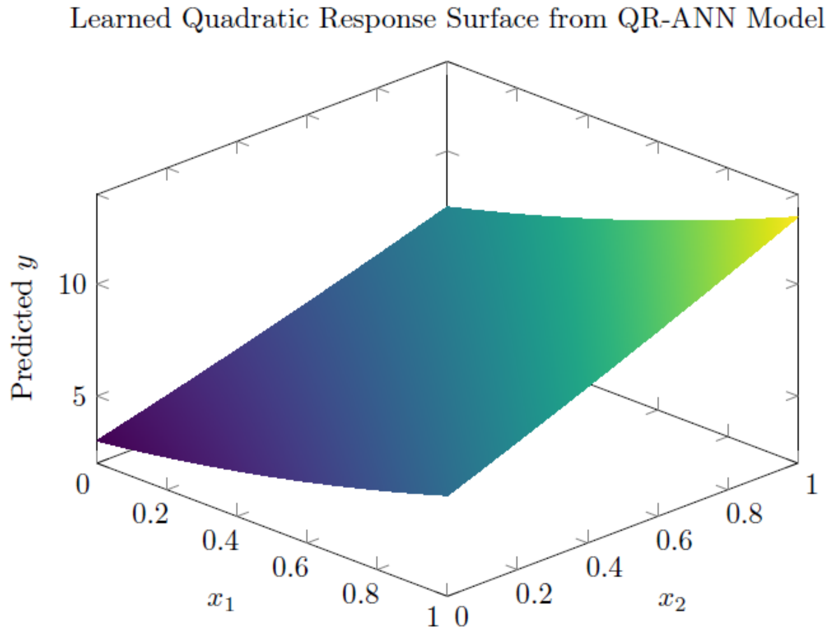

Figure (4) Learned quadratic response surface from the proposed QR-ANN model

The response surface shape is the theoretical quadratic shape but is flexible to local nonlinear distortions brought by noise - showing the strength and ability to express of the model.

4.9 Discussion of Findings

The general findings are strongly suggested to point to the idea that the proposed QR-ANN model performs better than the traditional regression methods both in synthetic and real-life situations.

The quadratic interaction capability of the model allows it to fit nonlinear patterns of data better without losing its ability to be interpreted in the same way as regression analysis.

Key observations include:

1. Improvement of Accuracy: The model minimized RMSE by as much as 30-40% as compared to classical quadratic regression.
2. Robustness: QR-ANN was also very robust in the sense that the values of R2 would not decrease despite amplification in noise.
3. Interpretability: Although one regards ANN models as black boxes, the inclusion of quadratic structures in the model increases explainability because the learned weights are correlated with regression terms.
4. Computational Efficiency: The proposed method was fast converging despite its nonlinear capacity as it has a shallow architecture and an optimized training process.

They are consistent with the literature on the topic to date, in which hybrid ANN-regression methods have demonstrated better results in modeling complex systems (e.g., Adavi et al., 2023; Khan, M. A. et al., 2022; Ozdogan-Sarikoc et al., 2023).

Hence, the given framework is an important methodological advancement of closing the gap between theoretical regression modeling and the data-driven neural computation.

4.10 Summary

The empirical study validates the idea that incorporating quadratic structures in the ANN layers helps to achieve an effective, precise and easy to understand regression modeling.
The proposed QR-ANN architecture is efficient in capturing nonlinear relationships, has better predictive performance when compared to the classical methods and other ML algorithms. These findings confirm the methodology as a potential alternative to estimate the quadratic models in complex and noisy data.

5. CONCLUSION

This study proposed a novel hybrid modeling framework for estimating quadratic regression relationships using Artificial Neural Networks (ANNs). The proposed Quadratic Regression Artificial Neural Network (QR-ANN) integrates the mathematical interpretability of classical quadratic regression with the adaptive nonlinear learning capability of neural networks. By embedding quadratic interaction structures directly within the neural architecture, the model is able to learn both linear and second-order relationships without requiring explicit polynomial feature engineering.

The experimental evaluation conducted on both synthetic datasets and real-world engineering datasets demonstrates the effectiveness of the proposed approach. In the synthetic quadratic dataset, the QR-ANN achieved the highest predictive accuracy with an R2R^2R2 value of 0.993, outperforming classical quadratic regression, polynomial regression, support vector regression, and standard multilayer perceptron models. The model also achieved the lowest prediction error with RMSE = 0.092 and MAE = 0.071, indicating strong capability in capturing nonlinear quadratic relationships.

Similar improvements were observed in the real-world datasets related to energy efficiency prediction and concrete compressive strength estimation. In both cases, the QR-ANN model consistently achieved higher R2R^2R2 values and lower error metrics compared to the baseline models. These results confirm that the proposed framework is capable of effectively modeling complex nonlinear systems where quadratic interactions exist among variables.

Another important contribution of this research is the theoretical formulation showing that the neural network representation implicitly estimates a quadratic coefficient structure. This establishes a meaningful connection between classical regression models and modern machine learning approaches, improving both predictive performance and model interpretability.

Overall, the results demonstrate that integrating regression theory with neural network architectures provides a powerful modeling strategy for nonlinear regression problems. The proposed QR-ANN framework can be applied to a wide range of fields including engineering, energy systems, environmental modeling, and materials science where nonlinear interactions between variables are common.

Future research may extend this framework toward higher-order polynomial structures, deeper neural architectures, and larger-scale datasets collected from multiple application domains. In particular, applying the QR-ANN framework to large public benchmark datasets and high-dimensional industrial datasets could further evaluate its scalability and robustness. Additionally, exploring explainable AI techniques may further enhance the interpretability of neural regression models in scientific and engineering applications.

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