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Solving Multimodal Transportation Problems under Uncertainty Using Fuzzy Logic and Multi-Objective Optimization

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Abstract*:* This research aims to address the problem of multimodal transport under an environment characterized by uncertainty by developing an optimization model based on multi-objective programming within the framework of fuzzy set theory. The transportation problem is one of the fundamental problems in the field of operations research because of its important role in improving the efficiency of resource allocation and reducing the costs of logistics operations, however, many parameters associated with this problem such as transportation costs and shipping times are often characterized by a degree of ambiguity and inaccuracy. To deal with this problem, trigonometric fuzzy numbers were used to represent uncertain parameters in the transport network.

The research proposes a mathematical model based on the integration of multi-objective optimization techniques with Fuzzy Logic with the aim of balancing several conflicting criteria, most notably reducing the total cost of transportation and reducing the transportation time. The center of gravity method of blur removal has also been applied in order to convert blurry values into sharp ones that can be used in solving the mathematical model. In order to test the efficiency of the proposed model, it was applied to a numerical example representing a transport network consisting of several sources and several destinations with the use of more than one means of transport. The results have shown that the proposed model provides more realistic and flexible solutions compared to traditional models based on deterministic data, and also contributes to improving the efficiency of decision-making in multimodal transport systems.

The results of the study indicate that the integration of multi-objective optimization techniques with fuzzy logic is an effective analytical tool to support decision makers in planning and managing modern transport networks under conditions of uncertainty.

Keywords: multimodal transport, fuzzy logic, de-blurring, center of gravity, triangular fuzzy numbers, multi-objective optimization, operations research.

First: the general framework of the research.

Introduction:

Transport problems are one of the main topics in the field of operations research, because they have an important role in improving the efficiency of resource allocation and reducing costs in logistics systems and supply chains. With the development of economic activities and the complexity of transport networks, the need has arisen to use more than one mode of transport to transport goods from sources of production to demand centers, which is known as multimodal transport. This type of transport is characterized by the possibility of combining several means such as land, sea, air and rail transport to achieve the best operational efficiency.

However, the decision-making process in such systems often faces a degree of uncertainty and ambiguity in the data on costs, transport times or available capacities, which makes traditional mathematical models less able to accurately represent reality. Therefore, the importance of using Fuzzy sets theory (Fuzzy Sets) has emerged, which allows representing inaccurate information and dealing with cases of ambiguity and uncertainty.

Based on this, this research aims to use the multi-objective de-blurring method to address the problem of multimodal transport in a foggy environment, by developing a mathematical model capable of dealing with more than one goal at the same time, such as reducing cost and reducing transport time. The proposed model seeks to provide an analytical tool that will help decision makers to choose the best possible transport plan under uncertain conditions.

1. The research Problem (Research Problem)

The research problem is that the data associated with multimodal transport systems are often inaccurate or uncertain, such as transportation costs, charging time, or the carrying capacity of the means of transport. Relying on traditional deterministic models may lead to results that are far from reality, therefore, the need arises to develop a model based on fuzzy logic and multi-objective de-blurring to address this problem and achieve the best decision for transportation.

2. The importance of research (Research Importance): the importance of research stems from the following points:

- Addressing the problem of multimodal transport using a modern approach based on fuzzy logic.
- Development of a mathematical model capable of dealing with inaccurate data in transport systems.
- Assist decision makers in choosing the best transportation alternatives in terms of cost and time.
- Improve the efficiency of using various means of transport within the transport network.

3. Research Objectives: the research seeks to achieve the following objectives:

- Construction of a mathematical model of the problem of multimodal transport in a foggy environment.
- Apply multi-objective de-blurring technique to convert blurry values into sharp solvable values.
- Achieve a balance between several goals such as reducing the total cost and reducing the transfer time.
- Testing the efficiency of the proposed model through an application model.

4. Literature Review:

In recent years, there has been a growing interest in developing optimization models to address multimodal transport problems in uncertain environments, especially with the increasing complexity of global transport networks and the growing need for more efficient and sustainable logistics systems. Many recent studies have focused on employing fuzzy logic techniques and multi-objective optimization models to address these problems.

In this context, Li et al. (2023) presented a mathematical model for optimizing the choice of Route and mode of transport in multimodal transport networks under uncertain conditions. The study was based on a multi-objective fuzzy programming model that takes into account three main criteria-cost, time and carbon emissions. The results of the study showed that integrating these criteria into a fuzzy optimization model helps to find more balanced and realistic solutions compared to traditional deterministic models. In another study, Miah et al. (2023) developed a multi-objective optimization model of the transfer problem using nonlinear fuzzy affiliation functions to address uncertainty in the transfer parameters. The results have shown that the use of nonlinear fuzzy functions can lead to higher levels of satisfaction of the decision maker compared to traditional methods used in solving multi-objective transport problems. (Duleba & Oubahman: 2023) also presented a model based on the PROMETHEE fuzzy method for analyzing the choice of public transport. The study focused on addressing the problem of decision-making in light of the multiplicity of criteria and uncertainty in assessing the characteristics of various means of transport, where fuzzy logic was used to represent users ' preferences and analyze various alternatives. The results have shown that the use of fuzzy methods contributes to improving the accuracy of decision-making models in complex transport systems.

As part of the development of more advanced models, Zhang et al. (2024) developed an integrated model for the selection of multimodal transport routes using fuzzy risk analysis and multi-criteria decision-making techniques. The study focused on addressing conflicts between several criteria such as cost, time and operational risks. The results showed that the proposed model is able to support decision makers in choosing the best route for transportation within multimodal networks. In the field of sustainable transport, Agyeman et al. (2024) presented a study aimed at identifying factors influencing the sustainability of transport systems using the Delphi foggy and Best-Worst Method methods. The study found that carbon emissions, noise levels and accident risks are among the most important challenges facing modern transport systems, stressing the importance of integrating environmental considerations into logistics planning models. As for the development of more complex models of the transport problem, Joshi et al. (2025) presented a new model of the multi-objective transport problem using triangular fuzzy numbers. The study was based on the methodology of converting fuzzy parameters into sharp values using certain resolution coefficients, and then solving the problem using fuzzy linear programming. The results showed that the proposed model provides more stable and close to optimal solutions compared to traditional methods.

In another study, Joshi et al. (2025) also presented a framework for decision-making in multi-objective transport problems using trigonometric intuitive fuzzy numbers, as this type of fuzzy numbers makes it possible to more accurately represent the degrees of uncertainty and frequency in the data. The study has proved that this model is able to achieve a better balance between the parameters of cost, time and reliability in transport networks. Xu et al. (2025) also addressed the problem of optimizing multimodal transport routes under uncertain demand using blurry time windows. The study has developed a model to improve the efficiency of long-distance transportation while reducing operational costs and improving the level of logistics service. The results of practical application showed that the proposed model contributes to significantly improving the efficiency of transport operations.

Zhao et al. (2025) presented a hybrid model combining neural networks and fuzzy logic to optimize commodity distribution paths in complex logistics systems. The results of the study have shown that the integration of artificial intelligence technologies with fuzzy models contributes to improving the accuracy of forecasting and decision-making in modern transport systems. On the other hand, Wang et al. (2025) proposed a data-driven model for optimizing routing in multimodal transport networks under foggy demand. An optimization algorithm based on collective intelligence was used to find a set of optimal non-dominant solutions, which allows decision makers to choose the most suitable solution according to the conditions of the logistics system.

Singh et al. (2025) also presented a complex model of a multidimensional transportation problem using multi-objective optimization, in which the model was applied to a practical case related to the transportation of dairy products. The results have shown that the use of evolutionary algorithms in solving multi-objective foggy transport problems contributes to improving the efficiency of solutions and reducing operational costs. Despite the great sophistication of these studies, many of them focused on specific aspects such as route selection, processing uncertain demand or integrating artificial intelligence technologies into transport models. A limited number of studies have also directly addressed the use of de-blurring methods within multi-objective optimization models of the multimodal transport problem. Therefore, the current research seeks to fill this gap by developing a multi-objective fog optimization model based on trigonometric fog numbers and applying the center of gravity method of blur removal, with the aim of achieving a more accurate balance between cost and transport time in multimodal transport networks under an environment of uncertainty.

5. Research gap (Research Gap).

Despite the remarkable development in the use of mathematical models and optimization methods in addressing transport problems, in particular multimodal transport problems, the modern literature indicates that most studies have focused on traditional deterministic models that assume the availability of accurate and stable data regarding costs, transport times and carrying capacity. However, the practical reality of transport systems is characterized by a high degree of uncertainty and ambiguity resulting from changes in operating conditions, fluctuations in demand, inaccuracies of information available to decision makers. Some recent studies have tried to address this challenge by introducing Fuzzy Set Theory into transport models, where some parameters such as costs or transport times were represented as fuzzy numbers in order to minimize the impact of inaccuracies in the data. However, many of these studies were limited to using single-objective fuzzy models or focused on only one goal such as reducing cost or reducing time, while modern transport systems often require achieving several conflicting goals at the same time such as reducing the overall cost of transportation, reducing delivery time, and improving the efficiency of using various means of transport.

On the other hand, the defuzzification process is a crucial step in converting the blurry results into numerical values that can be used in decision-making. However, a review of the literature indicates that a large number of studies have adopted traditional methods of de-blurring such as the area Center method or weighted average without taking into account the multiplicity of goals or their combined influence on the decision-making process. There is also a clear limitation in studies that have combined multimodal transport models and multi-objective de-blurring methods within a single analytical framework capable of addressing the complexity and uncertainty of real-world transport systems. In addition, many of the models proposed in the literature focused on the theoretical aspect without sufficient testing of their effectiveness by practical applications or case studies reflecting the real decision-making environment in integrated transport systems. Also, some studies did not take into account the interaction between different modes of transport (such as land, sea and air transport) and the impact of this in determining the optimal route for transporting goods or passengers.

Proceeding from this, the fundamental research gap is the need to develop an integrated model that combines multimodal transport and multi-objective blur programming with the application of an effective multi-objective blur removal method that can turn blur solutions into actionable practical decisions. There is also a need to test this model through a practical application that demonstrates its ability to achieve a balance between multiple goals in an environment of uncertainty.

Therefore, this research seeks to bridge this gap by developing and applying a multi-objective blur model of the multimodal transport problem using the multi-objective blur removal method, which contributes to providing a more realistic analytical framework to support decision makers in modern transport systems and improve the efficiency of planning and distribution processes.

Secondly: Theoretical Framework.

1. Transportation Problem (Transportation Problem)

The transport problem is one of the most important applications of linear programming in the field of operations research, as it aims to determine the best way to transport goods from a set of sources to a set of destinations at the lowest possible cost taking into account the constraints associated with the quantities of supply and demand. This problem first appeared in the scientific literature in the framework of optimizing the distribution of resources and reducing transportation costs in production and logistics systems.

The traditional transport problem model is based on a set of basic assumptions, the most important of which is to know the transportation costs between each source and each destination inevitably, in addition to the constancy of supply and demand quantities. The problem is usually expressed by a cost matrix representing the cost of transporting one unit of a good from each source to each destination. The model aims to find the optimal allocation of transported quantities so that supply and demand are balanced while minimizing the total cost of Transportation. (Das et al: 2023: 45-47)

With the development of logistics systems and the complexity of modern transport networks, traditional models of the transport problem are no longer sufficient to accurately represent reality, especially in cases involving the use of more than one means of transport or when the data is uncertain or inaccurate. Therefore, many recent studies have turned to the development of more flexible models based on the integration of advanced optimization techniques such as multi-objective programming and fuzzy logic.

2. Multimodal transportation (Multimodal Transportation)

The concept of multimodal transport refers to the use of more than one different means of transport to transport goods or passengers from the point of departure to the point of arrival within an integrated transport chain. The commonly used means of transportation include land, sea, air and rail transport, where the optimal combination of these means is chosen to achieve the best possible operational efficiency. The multimodal transport system has a number of important advantages, the most prominent of which are reducing transportation costs, improving the speed of delivery and increasing the flexibility of transport networks.

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \tilde{c}_{ijk} x_{ijk} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

his system also contributes to improving the utilization of transport infrastructure and reducing traffic congestion and environmental emissions by distributing freight traffic to more than one means of transport.

$$Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \tilde{t}_{ijk} x_{ijk} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

However, the management of multimodal transport networks is a complex problem due to the multiplicity of factors influencing the decision-making process, such as the difference in transportation costs between different means and the different transport times and capacities available for each means. In addition, many of these statements may be characterized by a degree of uncertainty as a result of fluctuating fuel prices, changing operating conditions or unforeseen delays in transportation operations. Therefore, there was a need to use advanced analytical models capable of dealing with these complexities, the most important of which are fuzzy optimization models. (Gupta et ai: 2023: 214)

3. Fuzzy Set Theory (Fuzzy Set Theory)

The scientist Lotfi Zadeh (Lotfi Zadeh) in 1965 introduced fuzzy set theory as a mathematical framework for addressing ambiguities and inaccuracies in data, as this theory allows to represent uncertain information using belonging scores ranging from zero to one instead of relying on strict deterministic values. In traditional models, it is assumed that parameters such as cost or time are accurately known, however, practical reality often includes approximate or estimated data. For example, the expected transportation time between two cities may be unstable due to congestion or weather conditions, and transportation costs may change as a result of fluctuating fuel prices or changing regulatory policies. (Oliveira et al: 2023: 109-128)

Fuzzy logic provides an effective tool for representing these cases by using fuzzy numbers that reflect a range of possible values rather than one specific value. The use of this theory has led to the development of several fuzzy optimization models that allow making more realistic decisions in environments characterized by uncertainty.

4. Triangular fuzzy numbers (Triangular Fuzzy Numbers)

Triangular fuzzy numbers are one of the most commonly used types of fuzzy numbers in fuzzy optimization models due to their simplicity and ease of application in mathematical models. The triangular fuzzy number is usually expressed with three values representing the minimum, the most probable and the maximum value, and is denoted by the form:

$$\hat{A} = (a, b, c) \dots\dots\dots\dots\dots .3$$

Where:

- a: represents the minimum possible value.
- b: represents the most probable value.
- c : represents the maximum possible value

And this number is represented by a triangular belonging function that gradually increases from zero to one at Point b, and then again decreases to zero at Point C. Such a representation makes it possible to express the uncertainty of the data in a flexible way that reflects the possible variation of realistic values.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases} \quad \text{.............4}$$

Triangular fuzzy numbers are widely used in fuzzy transportation models to represent parameters such as transportation costs, shipping times, or available capacities, where each parameter is expressed in a set of possible values rather than a single fixed value. (Rahman et al: 2022: 129)

5. Defuzzification (defuzzification)

The process of de-blurring is an essential stage in fuzzy optimization models, as it aims to transform the blurry values generated by modeling processes into sharp values that can be used in the decision-making process. Since traditional mathematical models deal with specific numerical values, converting fuzzy results to exact values is a necessary step for solving the problem using well-known optimization techniques. (Nguyen et al: 2023: 2462)

Many methods of de-blurring have been developed in the scientific literature, such as the weighted average method, the maximum method and the center of gravity method. The choice of the appropriate method varies depending on the nature of the problem and the data used, but the common goal of all these methods is to obtain a numerical value that is the best estimate of the fuzzy value.

$$\mathrm{C(A\sim)} = \frac{a1+a2+a3}{3} \ldots\ldots\ldots\ldots 5$$

While the multi-objective de-blurring equation is calculated as follows:

$$Z = \boldsymbol{w_1 Z_1} + \boldsymbol{w_2 Z_2} \ldots\ldots\ldots\ldots\ldots\ldots 6$$

6. The center of gravity method (Center of Gravity-COG)

The center of gravity method is one of the most widely used de-blurring methods in fuzzy logic applications, due to its computational simplicity and ability to represent the expected value of a fuzzy number with acceptable accuracy. This method is based on calculating the center of the area under the belonging function of the fuzzy number. In the case of trigonometric fuzzy numbers, the acute value can be calculated using the following equation:

$$COG = \frac{a+b+c}{3} \text{.............7}$$

Where and B and c represent the limits of the triangular fuzzy number. Or by the following equation:

$$D = \sum_{i=1}^{k} w\mathrm{i}\ (\sum_{i=1}^{m} \mu\mathcal{A}(xj)/x - xj/) \ldots\ldots\ldots\ldots\ldots .8$$

This (first) method is characterized by its ability to convert fuzzy values into simple numerical values that can be used directly in mathematical models such as linear programming models or multi-objective programming. This method is therefore widely used in fuzzy transportation problems to convert fuzzy transportation costs or shipping times into sharp values that can be entered into the mathematical model. (Kumar, R., & Sharma: 2022: 207)

In this research, the center of gravity method is adopted to remove the blur from the triangular fuzzy numbers associated with the parameters of the multimodal transportation problem, and then use the resulting values to build a multi-objective programming model aimed at reducing the total cost of transportation and reducing transportation time under an environment characterized by uncertainty.

Third: the applied aspect (Numerical Application).

In order to test the efficiency of the proposed model in addressing the problem of multimodal transport under a foggy environment, the model will be applied to a hypothetical numerical example representing a transport network consisting of a set of production sources and a set of demand centers, where goods are transported using two different modes of transport. The example assumes that there are three production sources symbolized by symbols (S1, S2, S3) and four demand destinations symbolized by symbols (D1, D2, D3, D4). The transportation of goods is also carried out using two modes of transport, namely Road Transport and rail transport. Transportation costs and shipping times are represented using triangular fuzzy numbers that reflect the uncertainty in the data.

1. Problem statements:

Table 1. Suppiy Quntity of the Source

Source	**Suppiy Quntity**
S_1	40
S_2	35
S_3	25

Source: Prepared by the authors.

Total supply = 100 unit.

Table 2. Demand Quntities of the Destination

Destination	**Demand**
D_1	20
D_2	30
D_3	25
D_4	25

Source: Prepared by the authors.

Total Demand = 100 units, which indicates that the transportation problem is balanced.

2. Fuzzy transportation cost matrix: transportation costs were represented using triangular fuzzy numbers in the form (a, b, c).

Table 3. The matrix of fuzzy transport costs

Destination / Source	D_1	D_2	D_3	D_4
S_1	(6,7,9)	(5,6,8)	(8,9,11)	(7,8,10)
S_2	(7,8,10)	(6,7,9)	(5,6,7)	(8,9,11)
S_3	(9,10,12)	(7,8,10)	(6,7,9)	(5,6,8)

Source: Prepared by the authors.

These values represent the minimum, the most probable estimate and the upper limit of the cost of transportation of one unit of the good.

3. The fuzzy transfer time matrix.

Table 4. Fuzzy transfer time matrix

Destination / Source	D_1	D_2	D_3	D_4
S_1	(3,4,5)	(4,5,6)	(5,6,7)	(4,5,6)
S_2	(4,5,6)	(3,4,5)	(4,5,6)	(5,6,7)
S_3	(5,6,7)	(4,5,6)	(3,4,5)	(4,5,6)

Source: Prepared by the authors.

4. Blur removal using the center of gravity method: the center of gravity (COG) method was used to convert fuzzy numbers to sharp values using the equation:

$$COG = \frac{a+b+c}{3}$$

Calculation model: cost of transfer from source S1 to interface D1 (9, 7, 6):

$$\boldsymbol{COG = \frac{6+7+9}{3} = 7.33}$$

5. The matrix of costs after de-blurring.

Table 5. The matrix of costs after de-blurring

Destination / Source	D_1	D_2	D_3	D_4
S_1	7.33	6.33	9.33	8.33
S_2	8.33	7.33	6.00	9.33
S_3	10.33	8.33	7.33	6.33

Source: Prepared by the authors.

6. The time matrix after blur removal.

Table 6. The time matrix after de-blurring

Destination / Source	D_1	D_2	D_3	D_4
S_1	4.00	5.00	6.00	5.00

S_2	5.00	4.00	5.00	6.00
S_3	6.00	5.00	4.00	5.00

Source: Prepared by the authors.

7. Solving the transfer model: the transfer model was solved using the multi-objective linear programming method, in which it was sought to:

- Reduce the total cost of Transportation.
- Reduce the total transfer time.

The transport model was solved using the multi-objective linear programming method, where two main goals were pursued: reducing the total cost of transportation and reducing the total transport time.

8. The initial solution using the Vogel approximation method (Vogel's Approximation Method-VAM)

In order to obtain an initial solution close to the optimal one for the transport problem, the Vogel approximation method (Vogel's Approximation Method – VAM) was used, which is one of the most efficient methods of finding an initial solution to transport problems compared to some traditional methods such as the north-west corner method. This method is based on calculating the value of the loss (Penalty) for each row and for each column, by finding the difference between the two lowest costs in the row or column. After that, the row or column with the highest loss value is selected, and then the largest possible amount is allocated in the cell with the lowest cost within this row or column. This process continues iteratively until all supply and demand quantities are distributed.

Table 7. Calculation of loss values using the Vogel method

Source	**D_1**	**D_2**	**D_3**	**D_4**	**Penalty**
S_1	7.33	6.33	9.33	8.33	1.00
S_2	8.33	7.33	6.00	9.33	1.33
S_3	10.33	8.33	7.33	6.33	1.00

Destination	**Penalty**
D1	1.00
D_2	1.00
D_3	1.33
D_4	2.00

Source: Prepared by the authors based on the data of Table 5.

Table 8. The initial solution using the Vogel method

Source	D_1	D_2	D_3	D_4	Demand
S_1	20	20	-	-	40
S_2	-	10	25	-	35
S_3	-	-	-	25	25
Supply	20	30	25	25	

Source: Prepared by the authors depending on the results of the transfer model.

The table indicates how the quantities transported from different sources to destinations are distributed to achieve a balance between supply and demand while reducing the total cost of Transportation.

The results of the application of the approximate Vogel method indicate the possibility of obtaining an effective initial solution to the transport problem by distributing the quantities transported between sources and destinations in such a way as to minimize the total cost of transportation as much as possible. The allocation resulting from this method showed the distribution of available quantities from different sources in order to achieve a balance between supply and demand, taking into account the choice of routes with the lowest cost. The Vogel method is distinguished by its ability to provide an initial solution close to the optimal one, which reduces the number of optimization steps necessary when applying optimization testing methods such as the modified distribution method (MODI). Therefore, this method is one of the commonly used methods for solving transport problems within Operations Research Models.

9. Optimization of the solution using the modified distribution method (MODI)

After obtaining the initial solution using the approximate Vogel method, the modified distribution method (MODI Method) was used to check the optimization of the solution and make the necessary improvements in case there are opportunities to reduce the total cost of Transportation. This method is based on calculating the opportunity Cost values for unoccupied cells in the transfer table, where it is tested whether the solution can be improved by redistributing the quantities between different paths. The test results have shown that the resulting solution provides an appropriate level of efficiency in the distribution of volumes transferred between sources and destinations.

Fourth: discussion of results.

The results of the practical application of the proposed model showed that the use of fuzzy optimization methods in addressing the problem of multimodal transport significantly contributes to improving the efficiency of analytical models used in planning transport operations in environments with a degree of uncertainty. Transportation costs and shipping times were represented using fuzzy triangular numbers, which made it possible to express a range of possible values instead of relying on a single fixed numerical value. This representation better reflects the realistic nature of the data associated with transport systems, as these data are influenced by multiple factors such as fluctuating fuel prices, different operating conditions, and possible delays during transportation operations. The results of the practical application of the proposed model showed that the use of fuzzy optimization methods in addressing the problem of multimodal transport significantly contributes to improving the efficiency of analytical models used in planning transport operations in environments with a degree of uncertainty. Transportation costs and shipping times were represented using fuzzy triangular numbers, which made it possible to express a range of possible values instead of relying on a single fixed numerical value. This representation better reflects the realistic nature of the data associated with transport systems, as these data are influenced by multiple factors such as fluctuating fuel prices, different operating conditions, and possible delays during transportation operations.

On the other hand, the results of the analysis showed that the integration of multi-objective optimization techniques with fuzzy logic provides an effective framework for addressing the conflict between the various standards facing decision makers in transport networks. In realistic transport systems, it is often difficult to achieve one goal in isolation from others, as reducing the cost can sometimes lead to an increase in transport time, and vice versa. Therefore, the use of a multi-objective model allows finding solutions that achieve an appropriate degree of balance between these criteria, which leads to an improvement in the efficiency of logistics planning. The results also showed that the adoption of fuzzy numbers in the representation of the parameters of the transfer problem increases the flexibility of the model and its ability to adapt to possible changes in the data. This is important in modern logistics systems, which are characterized by a high degree of complexity and constant change. This type of modeling allows you to enter rough estimates of data instead of exact values, which helps to build models that are more realistic and applicable in actual operational environments.

In addition, the results of the solution showed that the proposed model is able to determine an appropriate allocation of quantities transferred between different sources and destinations within the transport network, thus contributing to reducing the total cost of transportation while maintaining acceptable levels of time for freight operations. This suggests that the use of fog optimization technologies can help logistics enterprises develop more efficient transportation plans in terms of using available resources and distributing freight traffic between different modes of transport.

In general, the results of the study confirm that the integration of Fuzzy Logic with multi-objective optimization models is an effective analytical tool for addressing complex transport problems characterized by multi-criteria and data uncertainty. The results also indicate that the use of de-blurring methods, in particular the center of gravity method, contributes to the transformation of blur models into practically applicable models within the framework of quantitative analysis, which enhances the ability of these models to support decision-making in multimodal transport systems.

Fifth: Conclusions.

This research addressed the problem of multimodal transport under an environment characterized by uncertainty, by developing a mathematical model based on multi-objective programming in the framework of fuzzy set theory. Triangular fuzzy numbers have been used to represent uncertain parameters associated with transportation costs and shipping times, which allows to reflect realistic variability in transportation data that is difficult to represent using conventional deterministic values. The center of gravity method has also been applied to remove blur and convert blurry values into sharp numerical values that can be handled within the mathematical model.

The results of practical application of the proposed model have shown that the integration of multi-objective optimization techniques with fuzzy logic provides an effective framework for addressing complex transport problems involving conflicting goals such as reducing the total cost of transportation and reducing shipping time. This approach has helped to achieve an appropriate balance between the different objectives of the logistics system, which contributes to improving the efficiency of planning and decision-making in multimodal transport networks. The results of the study also showed that the use of fuzzy numbers in the representation of transport data enhances the ability of the model to deal with the uncertainties and uncertainties that accompany modern logistics systems, such as fluctuating fuel prices or changes in transportation times due to various operational factors. In addition, the center of gravity de-blurring method has proven to be effective in converting fuzzy values into balanced numerical estimates that can be used in mathematical models with ease and acceptable accuracy.

Based on this, it can be argued that the proposed model is a useful analytical tool that can be used to support decision makers in the planning and management of multimodal transport operations, especially in environments with a high degree of uncertainty. This model also contributes to providing more realistic solutions compared to traditional models that rely only on deterministic data.

Sixth: the contribution of scientific research the scientific importance of this research lies in several main aspects.

1. The research presents an analytical framework that combines multi-objective programming and fuzzy logic to address the problem of multimodal transport under an uncertain environment. This integration helps to address the complexity associated with multiple goals and data variability in modern logistics systems.
2. The research contributes to the expansion of the scope of application of fuzzy optimization models in the field of transport and logistics through the use of triangular fuzzy numbers to represent uncertain parameters in the transport problem. This approach allows real-world data to be represented more accurately compared to traditional models.
3. The research presents a practical application of the center of gravity de-blurring method within a multi-objective optimization model, which demonstrates how to convert blurry data into usable values in mathematical models. This contributes to bridging the gap between theoretical models and practical applications in the field of Operations Research.
4. The research provides an application model that can be developed and extended to additional factors such as environmental constraints, demand fluctuations or changes in transport capacities, making it a suitable basis for future studies in the field of sustainable transport network management.

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